Thin-Element and Thick-Element Optimizations of Non-Scaling FFAG Lattices

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Thin-Element Model

The model has two components:

- (Physics) Magnetic elements represented by kicks
- (Geometry) Path lengths from "law of sines"

Three stage refinement of our understanding of this model:

- (September 2003) Under assumption of equal cell length and equal integrated quadrupole strengths, Pythagoras theorem implies the shortest path-length ranking F0D0, Doublet, Triplet
- (December 2003) Under the assumption of equal cell length and equal betatron tunes, a calculation to second order shows that the increased quadrupole strengths of the Doublet and Triplet (compared to F0D0) exactly compensate the geometric effect so that path-length performance of the three lattice types is identical (at this order).
- (March 2004) Under similar assumptions, but to third order in element strengths, not only are the strengths greater *but also* the strength splitting is greater for the Triplet (in particular) and the Doublet when compared against the F0D0; and this leads to a reversal of the path-length ranking Triplet, (Doublet, F0D0).
- However, the main advantage of Doublet (over F0D0) is that the 2nd drift space may be very short, leading to reduced time-of-flight τ_0.
- However, fixed cell length is a false constraint; also magnets in the triplet tend to be longer.

Elaboration of the model

A simple example of the thin-element model: F0D0 with D-sector & F-quad.
Integrated strengths $\beta_d = B_{1d} l_d$ and $\beta_f = B_{1f} l_f$
Physics: equate angular kicks ψ to bending strengths.

$$\psi_d = (p_c\theta - \beta_d x_d)/p \ , \qquad \psi_f = \beta_f x_f/p \ , \qquad \psi_d + \psi_f = \theta \ . \tag{1}$$

Geometry: "law of sines"

$$\frac{(l_0 + \delta l_0)}{\sin\theta} = \frac{(x_f + l_0 \cot\theta)}{\cos\psi_d} = \frac{(x_d + l_0 \csc\theta)}{\cos\psi_f} \ . \tag{2}$$

Displacements (linearized) of closed orbits at D and F elements:

$$x_d \approx \frac{(p - \beta_f l)(p - p_c)\theta}{\beta_d\beta_f l + p\Delta\beta} \ , \qquad \text{and} \qquad x_f \approx \frac{p(p - p_c)\theta}{\beta_d\beta_f l + p\Delta\beta} \ . \tag{3}$$

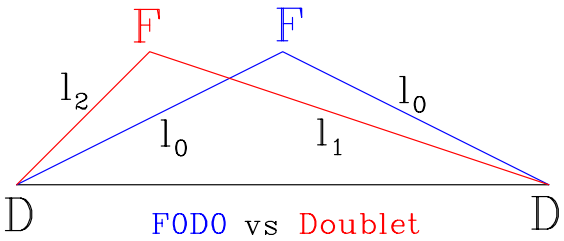
Equal strengths ($\beta_d = \beta_f$).

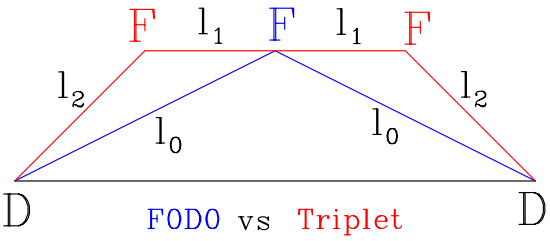
$$x_d = \frac{(p - \beta_f l)(p - p_c)}{\beta_f^2 l}\theta \ , \qquad x_f \approx \frac{p(p - p_c)}{\beta_f^2 l}\theta \ . \tag{4}$$

Here $l = l_0, l_2, \lambda$ for F0D0, Triplet and Doublet respectively, with $\lambda = l_1 l_2/l_0$.

- These formulae were known at the time of the *October* 2003 workshop.
- They are global in momentum description of behaviour; not merely a local linear indication as is given by argument for "minimum dispersion lattice".
- Formulae derived from Eq.3 show how to adjust reference momentum p_c so that *path lengths are equal* at injection and extraction momentum.
- Koscielniak pointed out that a strength splitting ($\Delta\beta = \beta_f - \beta_d > 0$) could be beneficial due to increase of size of denominator in (Eq.3), and that parabolae are not sacrosanct.
- What was *unknown* were the constraints from lattice tunes on the strengths β_d, β_f.

Geometry effect





Element strengths versus tunes

(December 2003) Starting from exact thick-element formulae for the tunes, expansions in powers of

$$A_{d,f} \equiv \frac{\beta_{d,f}}{p} = \frac{\text{integrated element strength}}{\text{momentum}} \tag{5}$$

were made to second order. Leading to predictions:

- $A^2 \times l = (1 - \cos\Phi)/\mathcal{L}_0$
- For equal tunes $\beta_d = \beta_f$ and $\beta_f^2 \times l = C$ where $l = l_0, l_2, \lambda$ for F0D0, Triplet, Doublet.
- Formula for minimum number of cells in an optimized lattice. Typically a slight under estimate $(1-2\%)$ for F0D0 and Doublet; and an over estimate $(10-15\%)$ for the Triplet.
- Lattice performance equal whether or not the F is a pure quadrupole, or a combined function magnet.

Requirement from W; and relative spread in cell transit times:

$$\frac{\delta p}{\omega\,\delta T}\frac{1}{(\hat{p}-\check{p})} \geq W\ , \qquad \text{and} \qquad \frac{\delta T}{\tau_0} = \frac{3}{4}\frac{(\hat{p}-\check{p})^2\,\theta^2}{(\check{p})^2(1-\cos\Phi)}\ . \tag{6}$$

Optimum number of cells N_c (after p_c is adjusted):

$$\frac{4}{3}\frac{(1-\cos\Phi)}{\omega\tau_0}\left[\frac{N_c}{\pi}\right]^2 \geq W\frac{(\hat{p}-\check{p})^3}{(\check{p})^2\delta p}\ . \tag{7}$$

W parametrizes longitudinal performance, and may be estimated either by particle tracking or from emittance-growth formulae.

δp is momentum impulse per cell. ω is RF angular frequency.
Φ is cell phase advance. θ is bend half-angle.

(March 2004) Continuing the expansion to third order one finds:

- $\beta_d \neq \beta_f$ for equal tunes.
- The splitting is largest and in the correct direction ($\beta_f > \beta_d$) for the Triplet.
- For equal tunes, the strength splitting is smallest and in the wrong direction ($\beta_d > \beta_f$) for the F0D0.
- Cololllaries:
  - For Triplet, the strong strength-splitting term in denominator will give a slight tilt to the parabola.
  - For the F0D0, a tune split between horizontal and vertical will increase the strength split and may have beneficial effect on path length.
- The expansion parameter is largest, and the truncated series approximation least accurate, at the injection momentum. But this is precisely the circumstance under which we specify the tune and design the lattice!
- The analogue of (7) for the optimum number of cells when $\beta_d \neq \beta_f$ may be obtained; and is typically an underestimate ($10 - 15\%$) for the Triplet.
- Strength splitting implies modification of the model longitudinal hamiltonian (mentioned at Oct. 2003 workshop).

The thin-element formulae for cells, strengths, closed orbits, pathlength, etc. form an excellent starting point for optimization of lattices to be designed using thick-element formulae.

Thick-Element Model and Formulae

Using *Mathematica*, exact, analytic formulae for the closed orbits, path length, tunes and betatron functions may be obtained for the very simple lattices (F0D0, Doublet, Triplet) considered for non-scaling FFAGs.

- These formulae can serve as basis for verification of computer codes.
- Assumption: magnetic elements are either sectoral combined function or parallel-faced quadrupoles with the entrance and exit faces perpendicular to the reference trajectory.
- Method depends on fact that for each momentum there is an arc-of-circle orbit through the sector(s) which is an exact solution of the equations of motion; and that an expansion may be made about each of these to match the orbit to the remainder of the lattice.
- The calculation has two components: (i) find the local arc orbits; and (ii) solve a matrix equation for the global closed orbit vector. Momentum is retained as a free variable throughout, so any matrices are parametrised by momentum.

Local arc orbits

Let momentum $p \equiv m_0 \gamma_u u$. The field is $B_z(\rho) = B_0 + B_1(\rho - \rho_c)$.
Equating momentum to bending, gives $(p - p_c) = (\rho - \rho_c)(B_0 + B_1\rho)$
with solution:

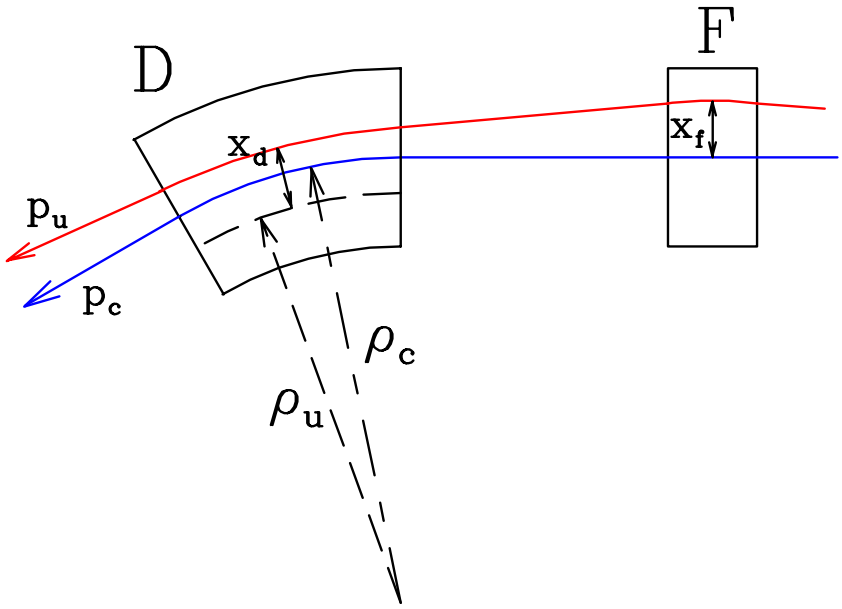
$$\rho_u \approx \rho_c \ + \frac{(p-p_c)}{\mathcal{B}} \ - \frac{B_1(p-p_c)^2}{(\mathcal{B})^3} + \ldots \qquad \mathcal{B} \equiv (B_0 + B_1\rho_c) \ . \tag{8}$$

- $\mathcal{B}$ determines whether the radii increase or decrease with momentum.
- If $\mathcal{B} > 0$, then the high momentum arcs are at larger radii than if the field were flat.
- If $\mathcal{B} < 0$, then arcs of higher momenta are at smaller radii than those of lower momenta.

- For good momentum compaction $|\mathcal{B}|$ should be as large as possible, i.e. $|B_1\rho_c| \gg |B_0|$. Here ρ_c is the bending radius at p_c.
- Formulae can be generalized to nonlinear field index.

Closed orbits

Simple example: F0D0 cell with D-sector and F-quad.



- l_0 is the drift length. l_f, l_d are F-quad and D-sector half-lengths. $k = \sqrt{B_1 c/p}$ where B_1 is the gradient. $\sigma = k \times l$.
- At entrance to the D sector, the coordinate jumps by an amount $\delta\mathbf{r} = (\delta\rho, 0)$ where $\delta\rho = (\rho_u - \rho_c)$ – because of difference between p_u and p_c coordinate systems.
- Transform the input vector $\mathbf{x}_0 = (x_0, 0)$ from the entrance of the half F quadrupole, to the exit of the half D sector. $\mathbf{D}, \mathbf{F}, \mathbf{O}$ are matrices.

$$(x, x') = \mathbf{D}_x(k_r l_d)\left[-\delta\mathbf{r} + \mathbf{O}(l_0) \cdot \mathbf{F}_x(k_f l_f) \cdot \mathbf{x}_0\right] . \tag{9}$$

- The value of x_0 that will make $x' = 0$ is the closed orbit.
- From this value, the displacement and divergence may be obtained at any other point in the cell by the appropriate matrix multiplication.

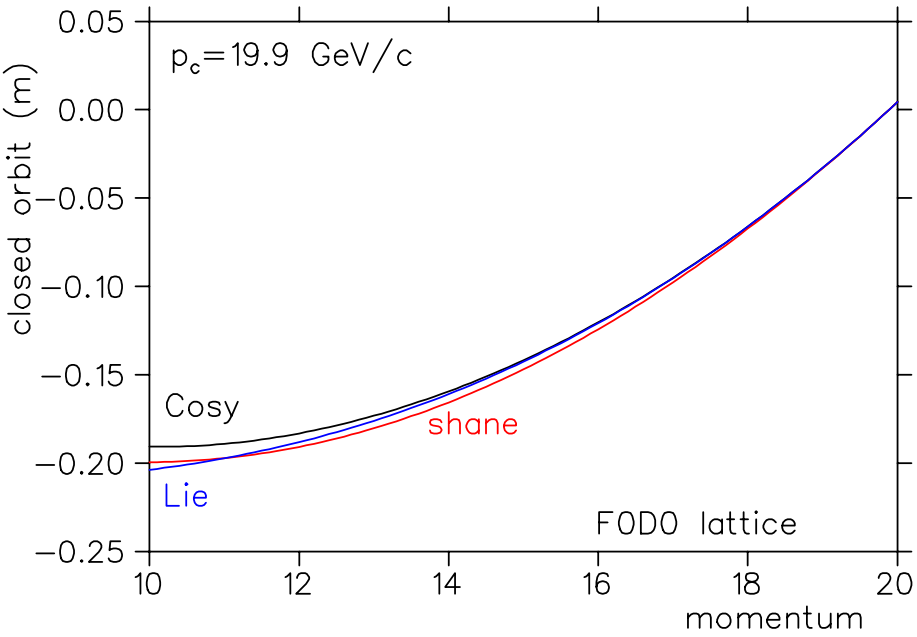
- At the centres of the full F and full D quadrupoles

$$x_f = (\rho_c - \rho_u)\mu_r \sinh\sigma_r/D \qquad \text{w.r.t. } \rho_c \tag{10}$$

$$x_d = (\rho_c - \rho_u)k_f\rho_c\omega_u \sin\sigma_f/D \qquad \text{w.r.t. } \rho_u \tag{11}$$

$$D = k_f\rho_c\omega_u \cosh\sigma_r \sin\sigma_f - \mu_r \sinh\sigma_r(\cos\sigma_f - k_f l_0 \sin\sigma_f)\ . \tag{12}$$

Working is increasingly more complex for Triplet and Doublet, but analgous expressions may be found; and from these path length may be obtained by Pythagoras theorem and integration.



Comparison of closed orbit predictions:
COSY vs Lie/Kaltchev vs analytic/Shane

Lattice Optimization

Assume the chief objective is to minimize the number of cells!

- The initial estimate of optimal parameters (such as N_c and p_c) for a given lattice type is based on the thin-element formulae.
- Initial guesses made for the thick element lengths must be consistent with keeping magnetic fields below desired limits.

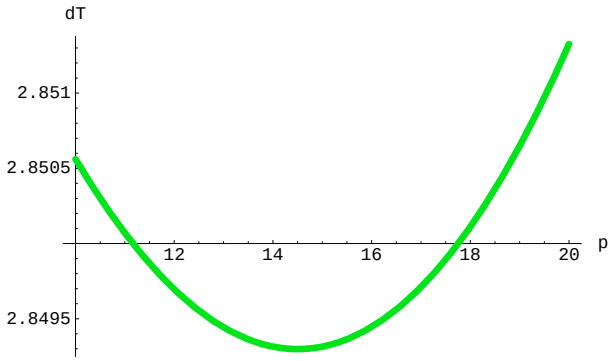
Five (5) free parameters:

- gradient of F element, B_{1f}
- gradient of D element, B_{1d}
- reference momentum, p_c
- length of the F sector, or quadrupole, l_f
- reverse bend angle of F sector, θ_f.

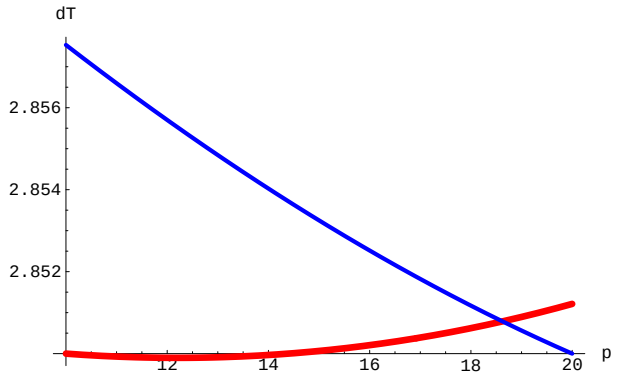
Fundamental constraint of *lattice closure*:
sum of bends is constant, $\theta_d + \theta_f = \pi/N_c$.

Additional constraints:

- Sum of magnetic element lengths $l_d + l_f = C$.
- Equal path lengths at injection and extraction momenta.
- Achieve given cell phase advances at injection momentum:
  $\Phi_x(\check{p}) = C_1$, $\Phi_y(\check{p}) = C_2$.
- Equal and opposite peak fields in D and F elements, $\hat{B}_d + \hat{B}_f = 0$.
- Equal and opposite F-displacements at injection and extraction momenta:
  $x_f(\check{p}) + x_f(\hat{p}) = 0$.



Pathlength as function of momentum before lattice adjustment.



Pathlengths at injection (red) and extraction (blue) momentum as function of reference momentum p_c.

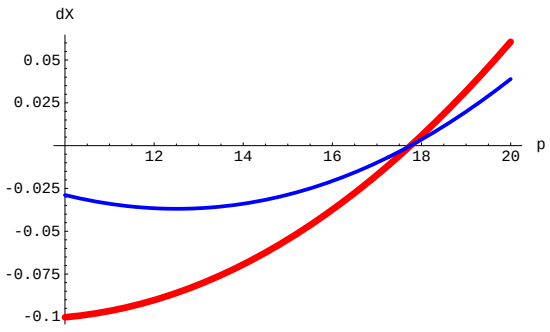
Reasons for these constraints

- Optimization at fixed cell length because W depends linearly on $\tau_0 = \mathcal{L}_0/c$.
- The pathlength variation from top to bottom of the parabola is minimized when $\delta\mathcal{L}(\check{p}) = \delta\mathcal{L}(\hat{p})$.
- Guess that betatron functions are minimized in both planes across all momenta when $C_1 = C_2$.
- Minimize peak magnetic fields.
- Centre beam in F-element – guess this minimimizes F-aperture.

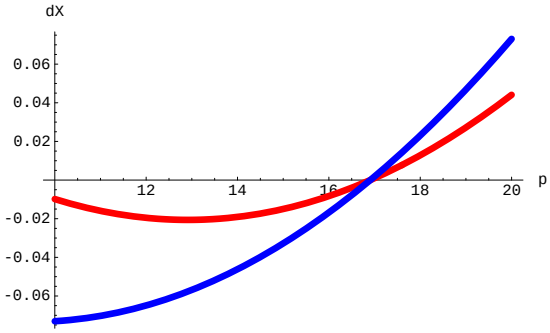
The last constraint (i.e. on x_f) is more arbitrary and deserves discussion.

- There is little difference in optical performance between, say F0D0, cells with one or two sector-type combined function magnets.
- What is different is the centring, or lack of it, within the F magnet.
- With a single sector bend (i.e. $\theta_f \equiv 0$), we have no choice in whether the low and high momentum orbits are disposed equally about the reference-momentum orbit.
- With two sector bends, we gain an additional free parameter, θ_f, and are at liberty to choose how this extra freedom is used.
  - e.g. Set the reference momentum at the minimum of the parabola; but in that case we have lost control of the disposition of the orbits.
  - e.g. If p_c "floats", then θ_f can be used to minimise one from several orbit quantities (e.g. centring and aperture). Because the F has the largest aperture (and dollar costs), we concentrate on the F sector.

Closed-orbit offsets at F-quad (red) and D-sector (blue) for simple F0D0 lattice.



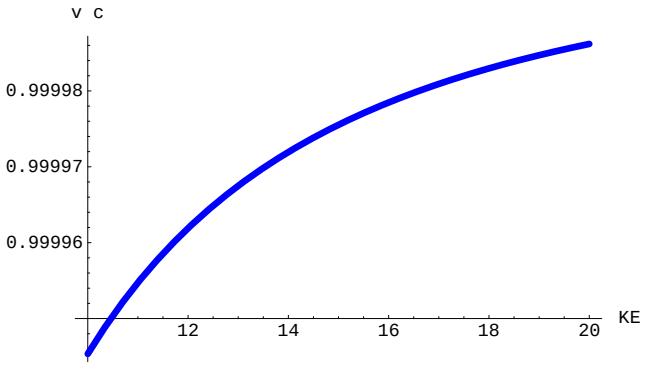
cell with D-sector & F-quad



cell with D-sector & F-sector

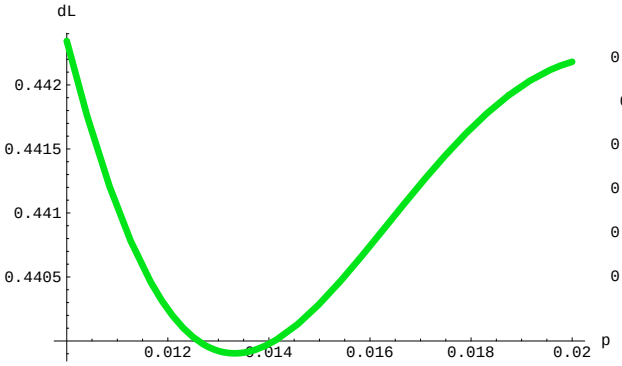
Results/Trends

- The optimization scheme produces "competitive" lattices.
- One can set target values for W, $\hat{B}_{d,f}$ and (usually) achieve them.
- Similar optical and pathlength performance for designs with "one sector and quadrupole(s)" or several sector magnets per cell.
- Slight advantage for F0D0 and Doublet with two sector magnets per cell.
- Slight advantage for Triplet with one sector and two quads per cell
  – perhaps need to change the "merit" for θ_f optimisation?
- For given targets, and lattice type, typically find several lattices with similar optical and pathlength properties but slightly different number of cells, magnet lengths and magnet apertures.
- Need dollar cost function to decide between these variations.
- Typically best pathlength performance is for Triplet-type lattice because greater element strengths and splittings (shorter cells?)
  – but usually requires also the longest magnets!
  – has steeper $\hat{B}$ cost function for symmetrizing parabola.
- Reiterate: need dollar cost function to decide between lattice types and high versus low field variants.
- At October 2003 workshop, relative pathlength variation of lattices was reduced to few parts in 10^4 over $\Delta p/p$ range of $\pm 33\%$
  – starts to be comparable with velocity variation! (few parts in 10^5).

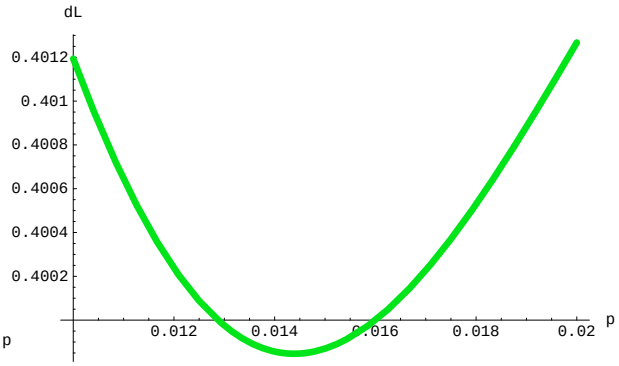


Small rings with few cells

- Electron model gives reason to consider machines with few cells and large bend angles.
- Strong? deviation from parabolic pathlength depedence is predicted by thin-element formulae for large θ.
- Will have a better model of the muon machine, if more cells used in the electron model; favours lower voltage per cell.



22 cell F0D0 lattice, 0.5 MV/cell, $p_c = 14.11$ MeV/c



36 cell F0D0 lattice, 0.25 MV/cell, $p_c = 16.00$ MeV/c

Design for cell phase advance of 0.7π radian and $W > 1/12 = 0.08333$ and $B \leq 7$ T
F0D0

cells #	$\mathcal{L}_0$ (m)	D→F (m)	F→D (m)	D&F (m)	l_d (m)	l_f (m)	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c GeV/c	apD cm	apF cm
87	5.2	2	2	1.2	0.810	0.390	31.64	62.44	5.444	0	6.60	11.6	.0772	18.287	6.39	14.98
87	5.1	2	2	1.1	0.743	0.357	34.92	69.32	5.919	0	7.14	11.3	.0790	18.253	6.19	14.67
88	5.2	2	2	1.2	0.809	0.391	31.63	62.45	5.582	0	6.53	11.33	.0790	18.288	6.31	14.81
88	5.1	2	2	1.1	0.743	0.357	34.91	69.33	5.852	0	7.063	11.07	.0809	18.255	6.12	14.50
86	5.1	2	2	1.1	0.743	0.357	34.93	69.31	5.988	0	7.228	11.59	.0772	18.251	6.26	14.85
86	5.2	2	2	1.2	0.809	0.391	31.65	62.43	5.507	0	6.683	11.87	.0754	18.285	6.47	15.16
88	5.0	2	2	1.0	0.676	0.324	38.85	77.63	6.415	0	7.701	10.81	.0828	18.219	5.92	14.20
85	5.0	2	2	1.0	0.676	0.324	38.88	77.59	6.641	0	7.973	11.59	.0772	18.213	6.13	14.72
84	4.9	1.9	1.9	1.1	0.742	0.358	36.54	72.19	6.143	0	7.437	11.709	.0764	18.267	6.21	14.63
86	4.9	1.9	1.9	1.1	0.742	0.358	36.52	72.22	5.999	0	7.264	11.167	.0802	18.271	6.06	14.28
88	4.9	1.9	1.9	1.1	0.742	0.358	36.54	72.19	6.143	0	7.437	11.709	.0765	18.267	6.21	14.62
82	4.7	1.8	1.9	1.1	0.741	0.359	38.31	75.34	6.305	0	7.661	11.820	.0757	18.285	6.15	14.39
86	4.7	1.8	1.9	1.1	0.741	0.359	38.27	75.39	6.012	0	7.304	10.738	.0834	18.292	5.86	13.70
88	4.7	1.8	1.8	1.1	0.741	0.359	38.25	75.41	5.876	0	7.137	10.254	.0873	18.296	5.73	13.38
88	4.75	1.8	1.8	1.15	0.774	0.375	36.36	71.45	5.630	0	6.858	10.384	.0862	18.313	5.83	12.53
90	4.7	1.8	1.8	1.1	0.371	0.179	38.23	75.44	5.745	0	6.978	9.7988	.0914	18.299	5.60	13.08
90	4.9	1.9	1.9	1.1	0.371	0.179	36.48	72.26	5.733	0	6.940	10.189	.0879	18.278	5.79	13.63
90	5.1	2	2	1.1	0.3715	0.1785	34.89	69.35	5.722	0	6.906	10.580	.0846	18.258	5.98	14.17
90	5.05	2	2	1.05	0.355	0.170	36.76	73.30	5.985	0	7.203	10.454	.0856	18.251	5.88	14.03
90	5.2	2	2	1.2	0.405	0.195	31.62	62.47	5.269	0	6.385	10.832	.0826	18.292	6.18	14.47
91	5.2	2	2	1.2	0.405	0.195	31.61	62.48	5.205	0	6.315	10.593	.0845	18.293	6.11	14.31

Triplet

cells #	$\mathcal{L}_0$ (m)	D→F (m)	F→D (m)	D&F (m)	l_d (m)	l_f (m)	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	$B_{\rm peak}$ (T)	dTime (ns)	W #	p_c GeV/c	apD cm	apF cm
70	4.2	2	0.5	1.2	0.381	0.219	63.75	132.4	7.65	0	10.15	7.78	.1151	19.476	4.39	8.31
70	4.5	2	0.5	1.5	0.475	0.275	47.66	97.08	6.15	0	8.31	8.65	.1035	19.534	4.83	9.21
70	4.7	2	0.5	1.7	0.538	0.312	40.30	81.11	5.44	0	7.44	9.25	.0968	19.567	5.44	9.82
70	4.8	2	0.5	1.8	0.569	0.331	37.29	74.61	5.15	0	7.08	9.55	.0937	19.581	5.66	10.13
70	4.9	2	0.5	1.9	.6005	.3495	34.63	68.89	4.885	0	6.752	9.858	.0908	19.595	5.89	10.45
75	4.7	2	0.5	1.7	0.538	0.312	40.26	81.16	5.08	0	6.942	8.04	.1113	19.573	5.07	9.15
72	4.8	2	0.5	1.8	0.569	0.331	37.27	74.63	5.01	0	6.88	9.84	.0992	19.584	5.50	9.84
72	4.75	2	0.5	1.75	0.554	0.321	38.73	77.78	5.14	0	7.05	9.69	.1008	19.577	5.40	9.69
72	4.77	2	0.5	1.77	0.560	0.325	38.14	76.49	5.09	0	6.98	9.75	.1002	19.580	5.44	9.75
71	4.8	2	0.5	1.8	0.569	0.331	37.28	74.62	5.08	0	6.98	9.99	.0965	19.583	5.58	9.90
72	4.9	2	0.5	1.9	0.601	0.349	34.61	68.91	4.749	0	6.563	9.311	.0962	19.598	5.72	10.15
72	5.0	2	0.5	2.0	0.632	0.368	32.25	63.84	4.517	0	6.276	9.603	.0932	19.610	5.93	10.46

Doublet

cells #	$\mathcal{L}_0$ (m)	D→F (m)	F→D (m)	D&F (m)	l_d (m)	l_f (m)	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	$B_{\rm peak}$ (T)	dTime (ns)	W #	p_c GeV/c	apD cm	apF cm
90	4.2	2	1	1.2	0.401	0.199	41.96	80.45	5.379	0	6.942	8.890	.1007	18.539	5.77	11.66
90	3.7	2	0.5	1.2	0.395	0.205	53.37	97.63	5.568	0	7.876	7.910	.1015	18.883	5.87	10.15
90	4	2	0.5	1.5	0.494	0.256	39.92	72.78	4.461	0	6.445	8.663	.1033	18.923	6.65	11.08
85	4.5	2	1	1.5	0.500	0.250	32.05	60.60	4.587	0	6.050	10.824	.0827	18.612	6.87	13.32
85	4	2	0.5	1.5	0.494	0.256	39.95	72.75	4.723	0	6.827	9.719	.0921	18.919	7.07	11.74
85	4	2	0.5	1.5	0.503	0.247	39.33	75.21	4.643	0	6.74/7.09	9.806	.0913	18.927	7.13	11.81
85	4	2	0.5	1.5	0.485	0.265	40.57	70.53	4.804	0	6.91/6.58	9.637	.0929	18.910	6.97	11.69
85	4.3	2	1	1.3	0.434	0.216	38.16	72.72	5.270	0	6.853	10.258	.0873	18.561	6.36	12.67
85	4.3	2	1	1.3	0.427	0.223	38.73	70.49	5.355	0	6.95/6.62	10.19	.0878	18.552	6.31	12.63
85	4.3	2	1	1.3	0.442	0.208	37.58	75.18	5.184	0	6.76/7.11	10.33	.0867	18.570	6.42	12.73
84	4.5	2	1	1.5	0.500	0.250	32.06	60.59	4.642	0	6.123	11.085	.08077	18.611	6.95	13.48
84	4	2	0.5	1.5	0.494	0.256	39.94	72.71	4.779	0	6.906	9.956	.0899	18.917	7.14	11.89
83	4	2	0.5	1.5	0.494	0.256	39.94	72.70	4.837	0	6.990	10.20	.0878	18.916	7.22	12.03

Design for cell phase advance of 0.7π radian and $W > 1/12 = 0.08333$ and $B \leq 4.5$ T
FODO

cells #	$\mathcal{L}_0$ (m)	D→F (m)	F→D (m)	D&F (m)	l_d (m)	l_f (m)	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	$B_{\rm peak}$ (T)	dTime (ns)	W #	p_c GeV/c	apD cm	apF cm
95	5.2	2	2	1.2	0.405	0.195	31.58	62.52	4.986	0	6.048	9.7145	.0967	18.298	5.84	13.69
95	5.4	2	2	1.4	0.471	0.229	26.48	51.80	4.299	0	5.270	10.166	.0881	18.358	6.22	14.26
95	5.6	2	2	1.6	0.537	0.263	22.68	43.88	3.783	0	4.683	10.617	.0843	18.412	6.59	14.82
96	5.6	2	2	1.6	0.537	0.263	22.68	43.89	3.744	0	4.634	10.396	.0861	18.413	6.52	14.66
96	5.7	2	2	1.7	.5697	.2803	21.12	40.66	3.533	0	4.394	10.616	.0843	18.438	6.71	14.94
100	5.2	2	2	1.2	0.404	0.195	31.55	62.55	4.737	0	5.745	8.7625	.1021	18.305	5.55	13.00
100	5.4	2	2	1.4	0.471	0.229	26.46	51.83	4.085	0	5.006	9.1701	.0966	18.364	5.90	13.53
100	5.6	2	2	1.6	0.537	0.263	22.66	43.90	3.594	0	4.448	9.5711	.0935	18.417	6.26	14.07
98	5.7	2	2	1.7	0.570	0.280	21.12	40.67	3.461	0	4.304	10.186	.0879	18.439	6.57	14.63

Triplet

cells #	$\mathcal{L}_0$ (m)	D→F (m)	F→D (m)	D&F (m)	l_d (m)	l_f (m)	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	$B_{\rm peak}$ (T)	dTime (ns)	W #	p_c GeV/c	apD cm	apF cm
75	5.0	2	0.5	2.0	0.632	0.368	32.23	63.87	4.336	0	6.023	8.842	.1013	19.614	5.69	10.02
75	5.2	2	0.5	2.2	0.694	0.406	28.21	55.32	3.951	0	5.544	9.386	.0954	19.636	6.11	10.62
75	5.4	2	0.5	2.4	0.757	0.443	24.93	48.42	3.629	0	5.142	9.937	.0901	19.656	6.53	11.22
78	5.4	2	0.5	2.4	0.757	0.443	24.92	48.43	3.489	0	4.943	9.181	.0975	19.659	6.28	10.78
78	5.6	2	0.5	2.6	.8193	.4807	22.21	42.78	3.227	0	4.613	9.696	.0923	19.676	6.69	11.36
78	5.7	2	0.5	2.7	.8504	.4996	21.02	40.32	3.110	0	4.446	9.956	.0899	19.684	6.90	11.66
80	5.4	2	0.5	2.4	0.757	0.443	24.91	48.44	3.402	0	4.818	8.723	.1026	19.661	6.12	10.50
80	5.6	2	0.5	2.6	0.819	0.481	22.20	42.79	3.146	0	4.497	9.213	.0972	19.678	6.52	11.07
85	5.3	2	0.5	2.3	0.726	0.424	26.45	51.74	3.337	0	4.701	7.504	.1193	19.655	5.56	9.61
85	5.4	2	0.5	2.4	0.757	0.443	24.89	48.46	3.201	0	4.533	7.719	.1160	19.665	5.75	9.87
85	5.45	2	0.5	2.45	0.773	0.452	24.17	46.94	3.138	0	4.453	7.827	.1144	19.669	5.85	10.01

Doublet

cells #	$\mathcal{L}_0$ (m)	D→F (m)	F→D (m)	D&F (m)	l_d (m)	l_f (m)	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c GeV/c	apD cm	apF cm
95	4	2	0.5	1.5	0.494	0.256	39.88	72.77	4.226	0	6.102	7.772	.1152	18.926	6.30	10.49
95	4.2	2	0.5	1.7	0.560	0.290	33.76	61.46	3.733	0	5.460	8.222	.1089	18.950	6.79	11.07
95	4.4	2	0.7	1.7	0.563	0.287	30.74	56.82	3.686	0	5.154	8.580	.1043	18.814	6.66	11.64
95	4.5	2	1	1.5	0.500	0.250	31.99	60.64	4.104	0	5.410	8.655	.1034	18.622	6.14	11.89
95	4.5	2	0.5	2	0.659	0.341	27.07	49.13	3.177	0	4.733	8.899	.1006	18.983	7.54	11.94
95	5	2	1	2	.6650	.3350	22.31	41.56	3.106	0	4.229	9.787	.0915	18.725	7.28	13.32
97	4.5	2	0.5	2	0.659	0.341	27.07	49.13	3.111	0	4.634	8.534	.1049	18.983	7.38	11.69
97	4.7	2	0.7	2	0.662	0.338	24.83	45.62	3.078	0	4.387	8.878	.1008	18.861	7.23	12.24
97	4.6	2	0.6	2	0.661	0.339	25.88	47.27	3.094	0	4.500	8.707	.1028	18.919	7.30	11.97
95	4.7	2	0.7	2	0.662	0.338	24.84	45.62	3.143	0	4.480	9.258	.0967	18.860	7.39	12.50
94	4.7	2	0.7	2	0.662	0.338	24.84	45.61	3.177	0	4.528	9.457	.0947	18.859	7.47	12.63
93	4.8	2	0.8	2	0.663	0.337	23.91	44.13	3.196	0	4.478	9.848	.0909	18.808	7.50	12.06
93	5	2	1	2	.6650	.3350	22.31	41.55	3.173	0	4.320	10.21	.0876	18.723	7.44	13.61

Redesign Muon with F sectors
F0D0, Triplet, Doublet

cells #	$\mathcal{L}_0$ (m)	D→F (m)	F→D (m)	D&F (m)	l_d (m)	l_f (m)	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c GeV/c	apD cm	apF cm
90	5.1	2	2	1.1	.3711	.1789	35.04	69.16	6.262	2.056	6.880	10.375	.08629	16.799	5.829	13.95
98	5.7	2	2	1.7	.5688	.2812	21.20	40.52	3.8507	1.3606	4.288	10.044	.08914	16.903	6.44	14.45
96	5.7	2	2	1.7	.5687	.2813	21.20	40.51	3.9305	1.389	4.376	10.463	.08556	16.899	6.57	14.75
70	4.9	2	0.5	1.9	.59895	.3510	35.01	68.47	6.0437	3.405	7.335	11.79	.07592	16.1958	6.40	11.48
72	5	2	0.5	2	.6301	.3699	32.58	63.45	5.600	3.156	6.791	11.434	.07830	16.2238	6.44	11.46
73	4.9	2	0.5	1.9	.5991	.3509	34.96	68.50	5.798	3.266	7.037	10.872	.08235	16.2164	6.15	11.01
74	4.9	2	0.5	1.9	.5991	.3509	34.95	68.52	5.721	3.222	6.943	10.589	.08455	16.223	6.06	10.86
78	5.6	2	0.5	2.6	.8165	.4835	22.40	42.49	4.040	2.285	4.887	11.190	.08001	16.326	7.16	12.35
80	5.6	2	0.5	2.6	.8166	.4834	22.39	42.50	3.939	2.228	4.766	10.649	.08407	16.335	6.99	11.94
80	5.7	2	0.5	2.7	.8475	.5025	21.20	40.06	3.802	2.152	4.598	10.876	.08232	16.344	7.19	12.22
81	5.7	2	0.5	2.7	.8476	.5024	21.19	40.07	3.755	2.125	4.542	10.614	.08434	16.348	7.10	12.06
82	5.7	2	0.5	2.7	.8476	.5024	21.19	20.07	3.710	2.100	4.487	10.362	.08639	16.352	7.02	11.92
83	4	2	0.5	1.5	.4888	.2612	40.45	71.31	5.746	2.574	6.785	9.9216	.09023	16.916	6.78	11.81
93	5	2	1	2	.6599	.3401	22.51	40.95	3.666	1.490	4.236	10.031	.08952	16.973	7.04	13.41

Design for $W > 1/4 = 0.250$ and cell phase advance of 0.7π and 0.5 MV per cavity.
F0D0

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	$B_{\rm peak}$ (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
25	45	15	15	15	4.88	2.62	3.351	5.599	.1534	0	.1962	12.92	.2156	17.856	2.18	4.90
30	45	15	15	15	4.92	2.58	3.271	5.678	.1285	0	.1637	8.935	.3119	18.089	1.80	4.00
30	42	15	15	12	3.945	2.055	4.299	7.506	.1586	0	.1984	8.230	.3385	17.916	1.62	3.74
27	45	15	15	15	4.90	2.60	3.314	5.634	.1424	0	.1819	11.07	.2517	17.967	2.01	4.50
27	43	15	15	13	4.25	2.25	3.956	6.743	.1629	0	.2055	10.47	.2660	17.843	1.87	4.30
27	43.5	15	15	13.5	4.413	2.337	3.777	6.434	.1572	0	.1990	10.62	.2623	17.876	1.91	4.35
27	44	15	15	14	4.5745	2.4255	3.611	6.147	.1519	0	.1929	10.77	.2586	17.908	1.94	4.40

Triplet

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	$B_{\rm peak}$ (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
20	40	15	5	15	4.609	2.891	5.487	9.555	.2124	0	.2974	11.56	.2409	18.687	1.94	3.64
22	40	15	5	15	4.639	2.861	5.399	9.646	.1934	0	.2688	9.392	.2966	18.836	1.72	3.22
22	42	15	5	17	5.257	3.243	4.539	8.052	.1714	0	.2415	10.07	.2766	18.920	1.88	3.45
22	44	15	5	19	5.873	3.626	3.880	6.835	.1540	0	.2196	10.76	.2588	18.990	2.03	3.67
22	45	15	5	20	6.182	3.818	3.606	6.331	.1466	0	.2103	11.11	.2507	19.019	2.11	3.78
23	40	15	5	15	4.651	2.849	5.362	9.684	.1851	0	.2565	8.526	.3268	18.894	1.63	3.05
23	45	15	5	20	6.196	3.804	3.585	6.353	.1402	0	.2006	10.92	.2761	19.068	2.00	3.58
23	46	15	5	21	6.5045	3.9955	3.343	5.903	.1337	0	.1924	10.41	.2676	19.095	2.08	3.69

Doublet

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	$B_{\rm peak}$ (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
24	40	15	5	20	6.494	3.506	3.184	5.321	.1246	0	.1918	13.26	.2102	18.541	2.90	4.62
24	35	15	5	15	4.862	2.638	4.760	7.913	.1650	0	.2447	11.42	.2439	18.371	2.38	4.03
25	35	15	5	15	4.869	2.631	4.737	7.934	.1585	0	.2345	10.49	.2655	18.419	2.27	3.85
25	40	15	5	20	6.503	3.497	3.172	5.333	.1197	0	.1838	12.18	.2288	18.580	2.77	4.41
25	37	15	5	17	5.522	2.977	3.986	6.689	.1403	0	.2108	11.17	.2495	18.492	2.47	4.07
26	35	15	5	15	4.876	2.624	4.717	7.953	.1526	0	.2252	9.673	.2880	18.460	2.18	3.68
26	40	15	5	20	6.511	3.489	3.160	5.344	.1152	0	.1764	11.22	.2483	18.614	2.65	4.22
26	37	15	5	17	5.530	2.970	3.970	6.704	.1351	0	.2024	10.29	.2707	18.530	2.36	3.89
26	38	15	5	18	5.857	3.143	3.666	6.194	.1277	0	.1928	10.60	.2628	18.560	2.46	4.00
26	38	15	6	17	5.538	2.962	3.789	6.415	.1343	0	.1961	10.52	.2649	18.450	2.33	3.99

Design for $W > 1/4 = 0.250$ and cell phase advance of 0.7π and 0.25 MV per cavity.

F0D0

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
36	40	15	15	10	3.3156	1.6844	5.233	9.490	.1578	0	.1942	5.361	.2599	17.973	1.24	2.92
36	43	16	16	11	3.646	1.854	4.435	8.037	.1437	0	.1771	5.772	.2414	17.994	1.34	3.14
36	41	15	15	11	3.643	1.857	4.673	8.454	.1441	0	.1785	5.522	.2523	18.038	1.29	2.99
36	42	15	15	12	3.9705	2.0295	4.212	7.597	.1327	0	.1654	5.681	.2452	18.097	1.34	3.06
37	45	15	15	15	4.951	2.549	3.206	5.746	.1044	0	.1326	5.822	.2393	18.257	1.45	3.19
37	40	15	15	10	3.3185	1.6815	5.218	9.506	.1536	0	.1889	5.071	.2748	17.997	1.20	2.83
37	42	15	15	12	3.974	2.026	4.201	7.609	.1291	0	.1609	5.373	.2593	18.118	1.30	2.97
38	47	16	16	15	4.959	2.541	3.042	5.490	.1014	0	.1280	5.740	.2427	18.234	1.45	3.24

Triplet

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
28	40	15	5	15	4.696	2.804	5.233	9.829	.1521	0	.2085	5.580	.2497	19.088	1.29	2.41
29	40	15	5	15	4.703	2.797	5.214	9.850	.1468	0	.2010	5.178	.2691	19.114	1.24	2.31
29	41	15	5	16	5.014	2.986	4.775	8.973	.1380	0	.1902	5.368	.2595	19.147	1.30	2.39

Doublet

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
31	35	15	5	15	4.901	2.599	4.642	8.027	.1283	0	.1877	6.717	.2074	18.605	1.79	3.02
33	35	15	5	15	4.909	2.591	4.621	8.048	.1206	0	.1760	5.903	.2360	18.644	1.67	2.81
33	33	15	5	13	4.253	2.247	5.599	9.754	.1387	0	.1991	5.526	.2521	18.580	1.53	2.65
33	34	15	5	14	4.581	2.419	5.072	8.835	.1290	0	.1867	5.714	.2438	18.613	1.60	2.73
34	35	15	5	15	4.912	2.588	4.611	8.058	.1171	0	.1706	5.551	.2510	18.660	1.62	2.72

Electron model Lattices with F-Sector magnets

F0D0

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
27	44	15	15	14	4.630	2.370	3.671	6.238	0.1604	.06076	.1767	8.096	0.3442	15.431	1.52	3.72
26	44	15	15	14	4.636	2.364	3.689	6.248	0.1650	.06311	.1812	8.394	0.3320	15.273	1.53	3.78
25	44	15	15	14	4.645	2.355	3.707	6.265	0.1694	.06565	.1856	8.640	0.3225	15.088	1.53	3.83
24	44	15	15	14	4.657	2.343	3.730	6.286	0.1744	.07040	.1899	8.646	0.3223	14.825	1.48	3.80
23	44	15	15	14	4.679	2.321	3.750	6.330	0.1787	.07539	.1932	8.443	0.3721	14.509	1.40	3.72
22	44	15	15	14	4.718	2.282	3.763	6.414	0.1817	.08117	.1947	7.866	0.3542	14.108	1.27	3.54
36	40	15	15	10	3.331	1.669	5.306	9.533	0.1692	.05857	.1857	4.617	0.3018	15.999	1.08	2.67
35	40	15	15	10	3.330	1.670	5.325	9.526	0.1734	.06024	.1902	4.827	0.2886	15.932	1.10	2.73
34	40	15	15	10	3.330	1.670	5.344	9.522	0.1779	.06200	.1949	5.045	0.2762	15.857	1.12	2.79

Triplet

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
23	43	15	5	18	5.820	3.180	4.254	7.794	0.1625	.09884	.1945	5.263	0.5294	13.857	1.25	2.45
23	46	15	5	21	6.734	3.766	3.427	6.116	0.1437	.08677	.1723	6.201	0.4493	14.070	1.48	2.80
29	41	15	5	16	5.082	2.918	4.927	9.055	0.1568	.08906	.1888	4.554	0.3059	14.853	1.13	2.20
28	41	15	5	16	5.091	2.909	4.944	9.072	0.1607	.09202	.1934	4.681	0.2976	14.712	1.14	2.24

Doublet

cells #	$\mathcal{L}_0$ cm	D→F cm	F→D cm	D&F cm	l_d cm	l_f cm	B_{1d} T/m	B_{1f} T/m	B_{0d} (T)	B_{0f} (T)	B_{peak} (T)	dTime (ns)	W #	p_c MeV/c	apD cm	apF cm
26	38	15	5	18	5.820	3.180	3.791	6.069	0.1453	.06771	.1715	7.955	0.3503	15.634	1.88	3.42
25	38	15	5	18	5.823	3.177	3.808	6.070	0.1499	.07036	.1767	8.319	0.3349	15.488	1.92	3.50
24	38	15	5	18	5.830	3.170	3.826	6.076	0.1545	.07322	.1819	8.657	0.3219	15.319	1.94	3.58
33	33	15	5	13	4.223	2.277	5.747	9.577	0.1586	.06965	.1851	4.661	0.2989	16.099	1.28	2.41
32	33	15	5	13	4.222	2.278	5.766	9.568	0.1631	.07179	.1902	4.894	0.2846	16.027	1.31	2.48