

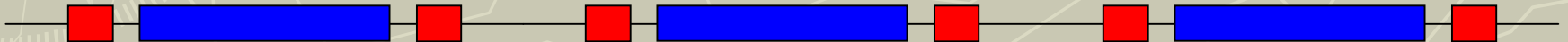
# 1.5-GeV Proton FFAG as Injector to the BNL-AGS

*Alessandro G. Ruggiero*

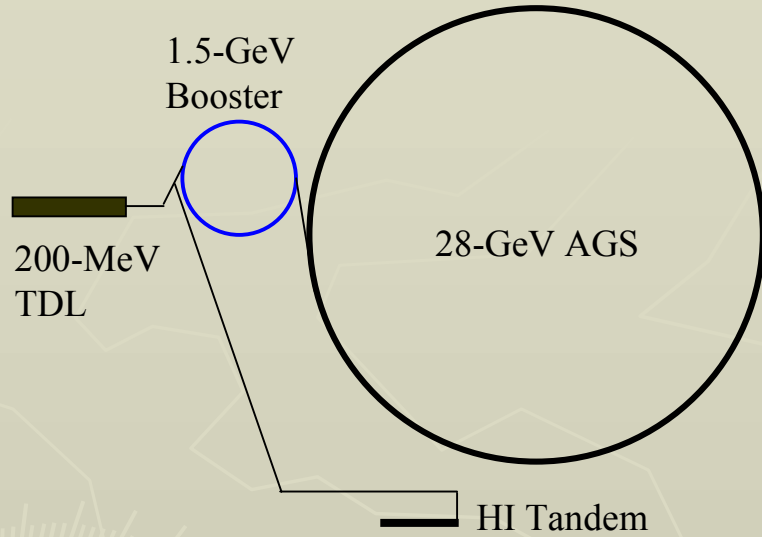
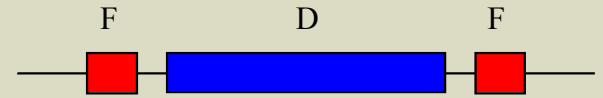
M. Blaskiewicz, E. Courant, D. Trbojevic,  
N. Tsoupas, W. Zhang

Brookhaven National Laboratory

FFAG 2004 Workshop, Vancouver, Canada. April 15-21 2004



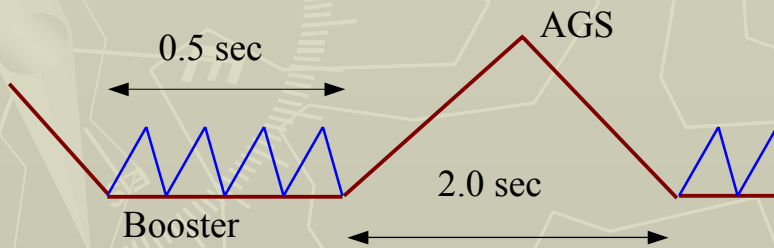
# Present BNL - AGS Facility



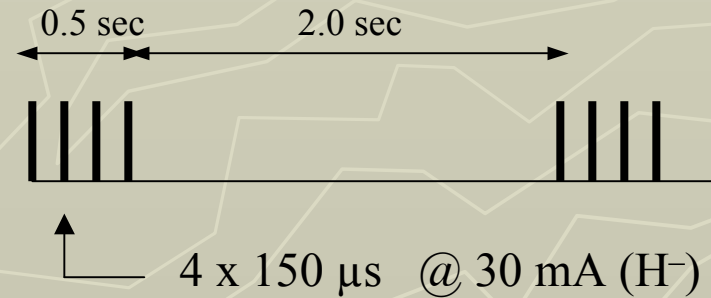
## Performance

Rep. Rate	0.4 Hz
Top Energy	28 GeV
Intensity	$7 \times 10^{13}$ ppp
Ave. Power	<b>125 kW</b>

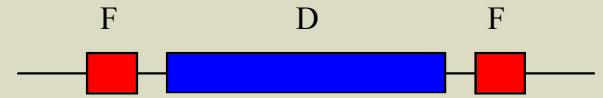
Typical AGS cycle for Protons



Typical DTL cycle for Protons



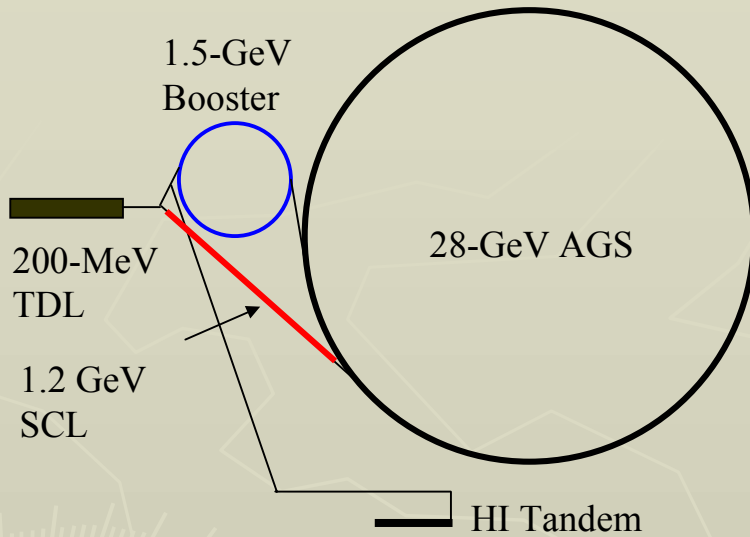
# AGS Upgrade with 1.2-GeV SCL



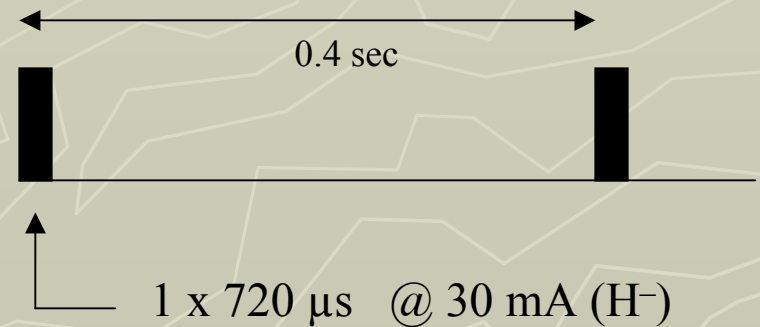
## Performance

Rep. Rate	2.5 Hz
Top Energy	28 GeV
Intensity	$10 \times 10^{13}$ ppp
Ave. Power	<b>1.0 MW</b>

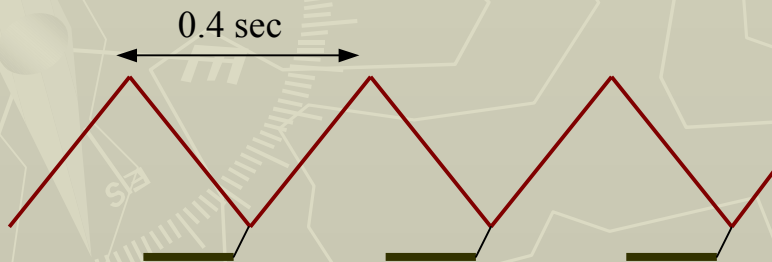
Only Protons, **no HI**



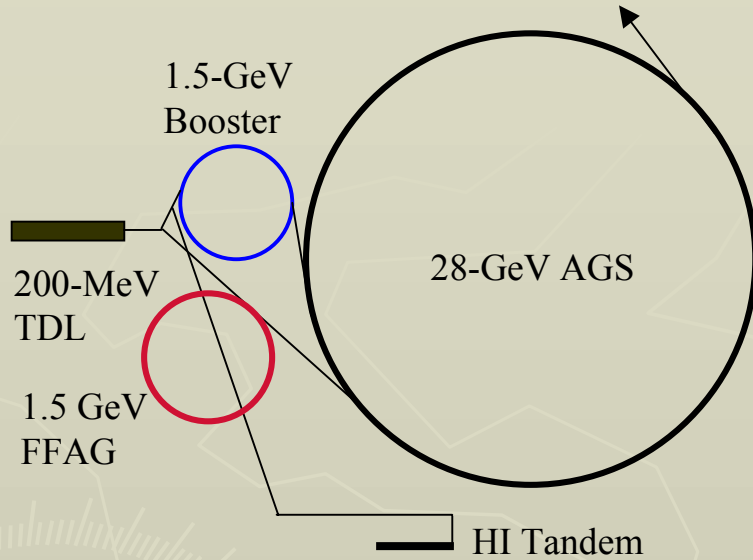
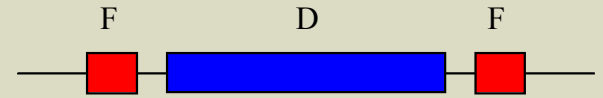
## Typical DTL cycle for Protons



## AGS Cycle with 1.2-GeV SCL



# AGS Upgrade with 1.5-GeV FFAG

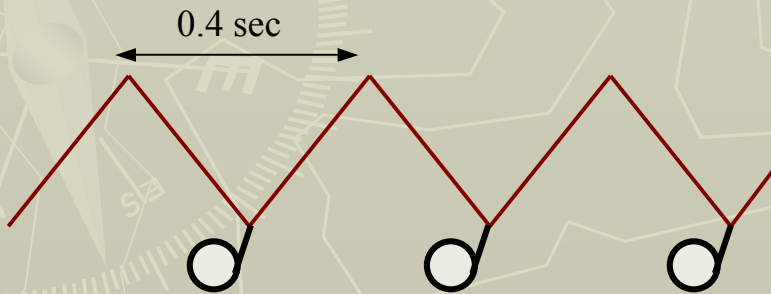


## Performance

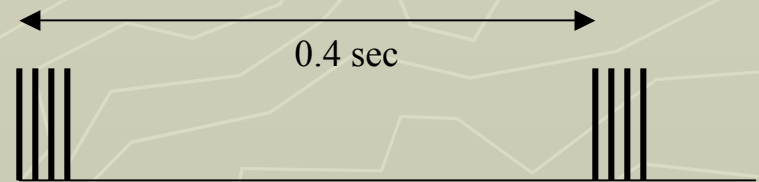
Rep. Rate	2.5 Hz
Top Energy	28 GeV
Intensity	$10 \times 10^{13}$ ppp
Ave. Power	<b>1.0 MW</b>

Only Protons, **and HI**

AGS Cycle with 1.5-GeV FFAG



Typical DTL cycle for Protons



4 x 135  $\mu$ s @ 35 mA  
1.5 ms apart (total 5 ms)

# 1.5-GeV AGS FFAG

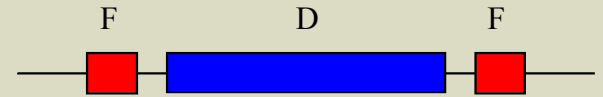


Table 1. Proton Beam Kinematic Parameters

	<u>Injection</u>	<u>Central</u>	<u>Extract.</u>
Kinetic Energy, MeV	<b>200</b>	<b>786.722</b>	<b>1,500</b>
$\beta$	0.5662	0.8391	0.9230
$\gamma$	1.213	1.838	2.599
Momentum, MeV/c	644.4	1,447	2,250
Magnetic Rigidity, kG-m	21.496	48.283	75.069
$\Delta p/p$	-0.5548	0.0	0.5548

Table 3. The AGS-FFAG Parameters for the Reference Trajectory

Circumference	244.439 m
Number of Periods	42
Period Length	5.81998 m
Short Drift, g	0.30 m
Long Drift, S	1.405954 m
Phase Advance / Period, H/V	97.7142° / 97.7142°
Betatron Tunes, H/V	11.40 / 11.40
Transition Energy, $\gamma_T$	39.7573 i

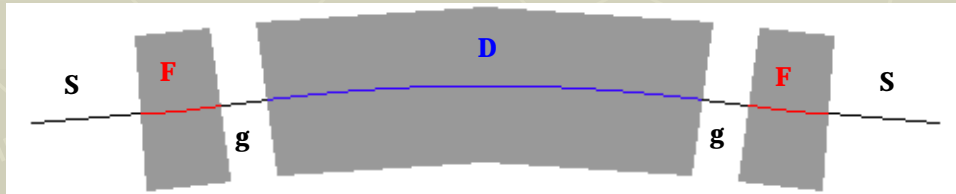
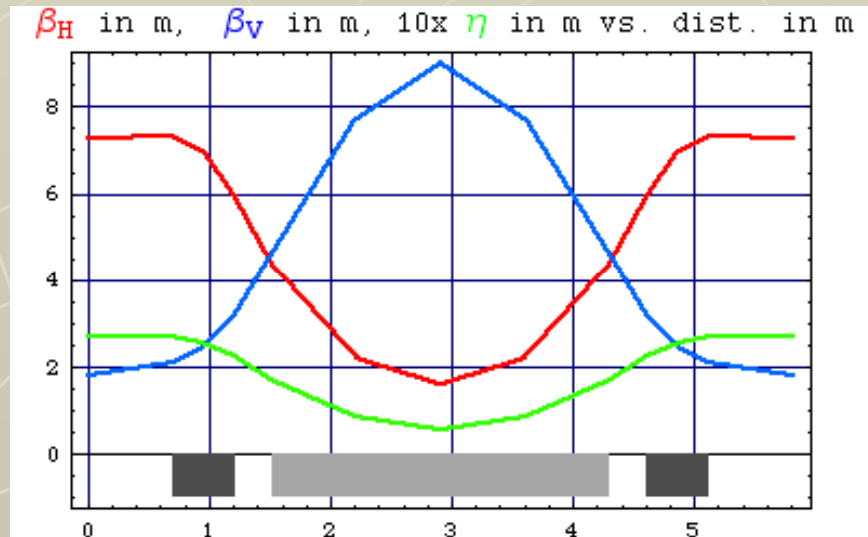


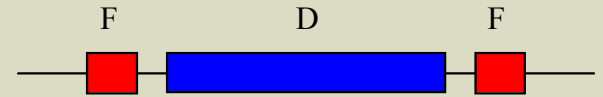
Table 2. Magnet Parameters

Magnet Type	<b>F</b>	<b>D</b>
Arc Length, m	0.509444	2.79514
Bending Field (B), kG	-5.29169	4.51305
Gradient (G), kG/m	33.9174	-12.4036
Field Index, $n = G/Bh$	74.4451	-37.4293
Bend Radius ( $\rho = 1/h$ ), m	-10.2943	12.0705
Bending Angle, mrad	-49.4877	231.569
Sagitta, cm	0.355537	9.12532

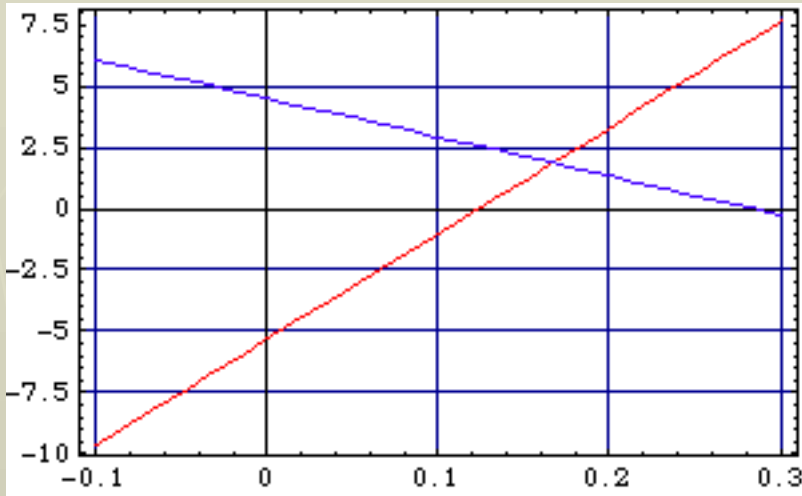
## Non-Scaling FDF Triplet



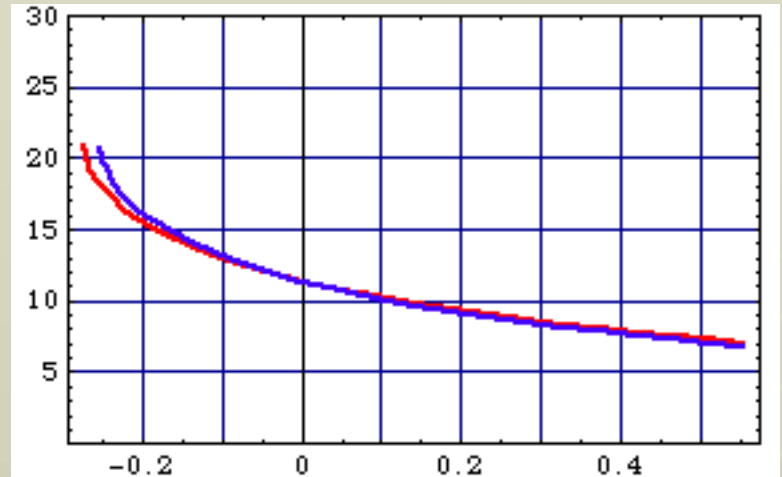
# Linear Field Profile (1 of 2)



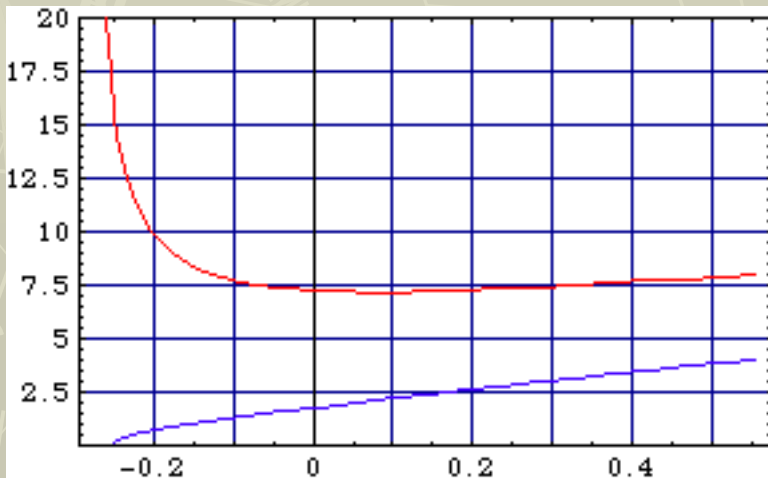
Field Profile (kG) vs. x (m) in  
 F-Sector Magnet (Red) D-Sector Magnet (Blue)



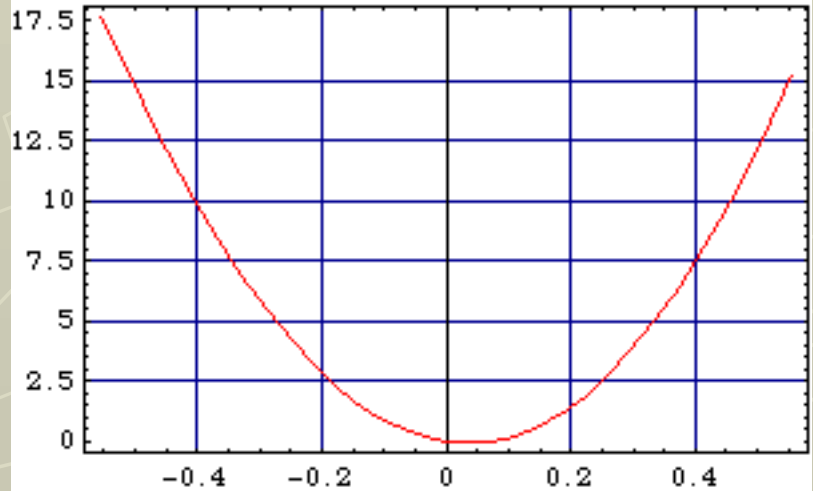
Tune variation vs.  $\delta$   $\nu_H$  (Red)  $\nu_V$  (Blue)



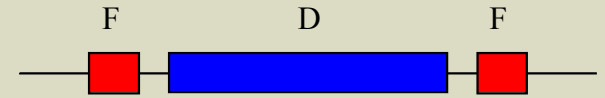
$\beta_H$  (Red) and  $\beta_V$  (Blue) in m vs.  $\delta$   
 at the beginning of a period (S)



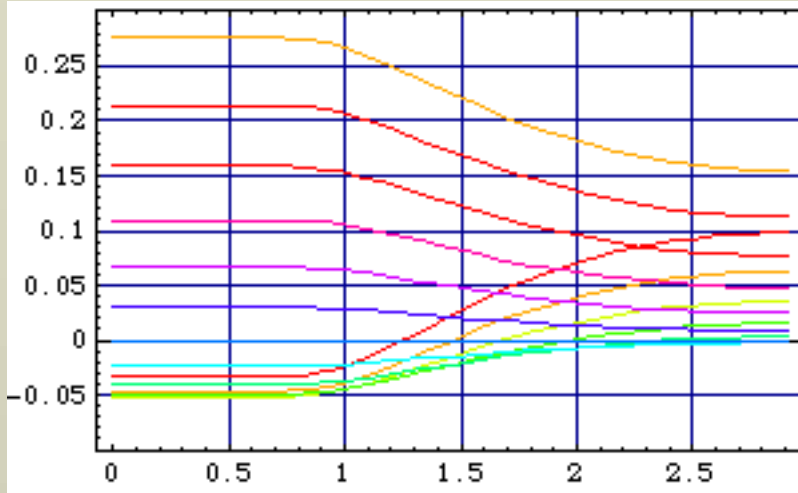
$\Delta L$  in mm / period vs.  $\delta$



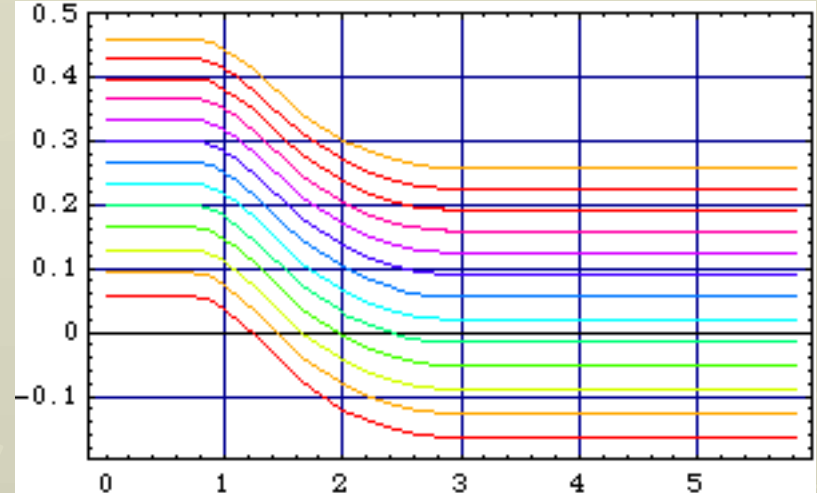
# Linear Field Profile (2 of 2)



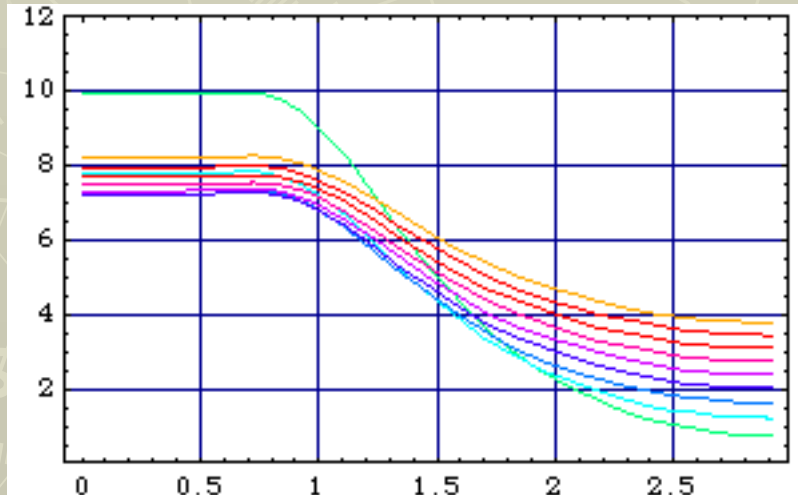
$x_{co}$  (m) vs. period length (m) for  $\delta = -0.6$  to  $+0.6$  in steps of 0.1



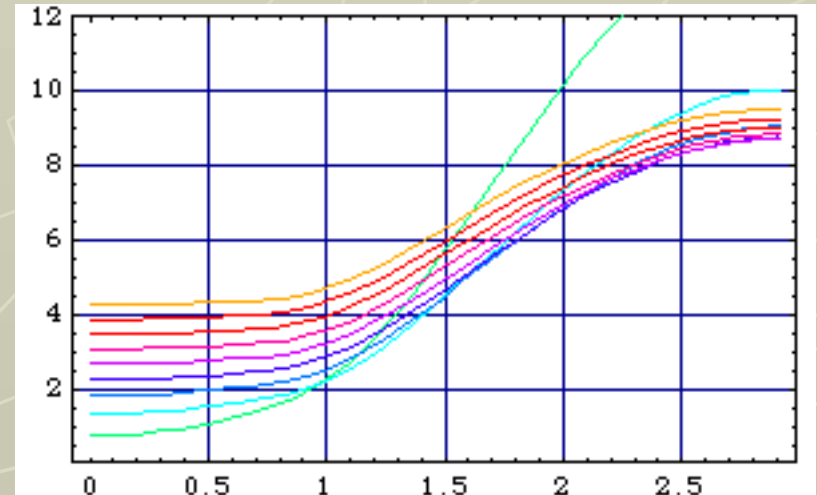
$\eta$  (m) vs. period length (m) for  $\delta = -0.6$  to  $+0.6$  in steps of 0.1



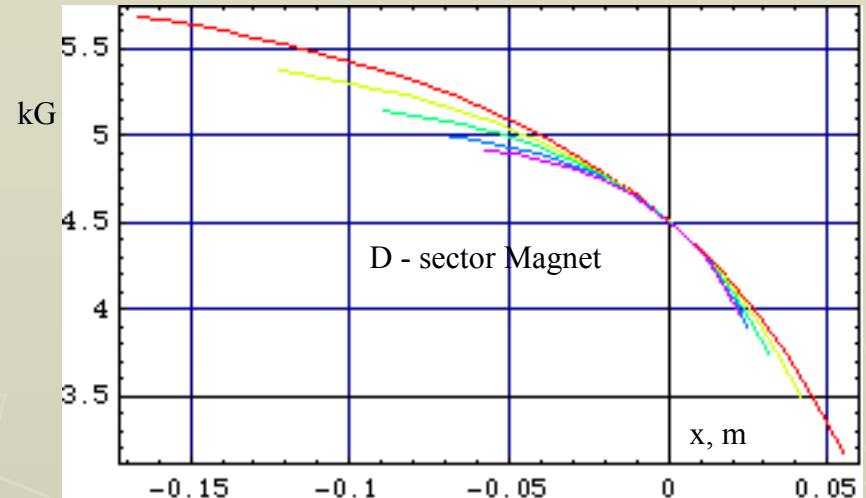
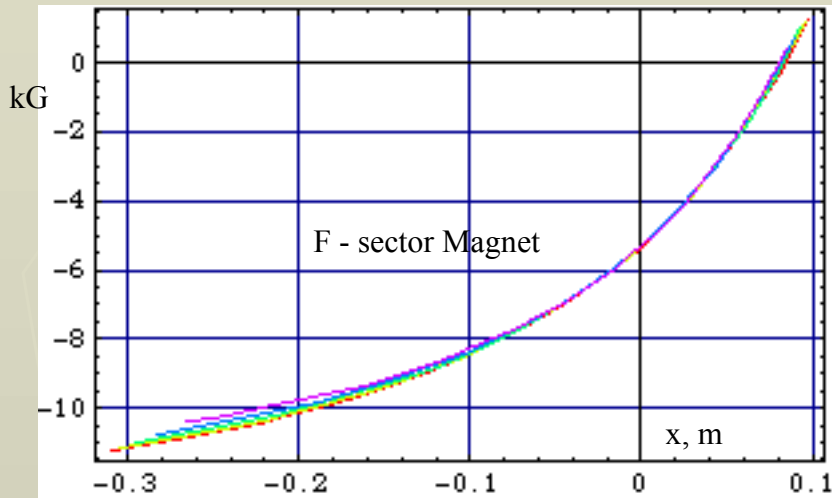
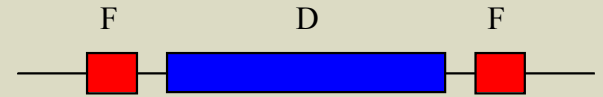
$\beta_H$  (m) vs. period length (m) for  $\delta = -0.6$  to  $+0.6$  in steps of 0.1



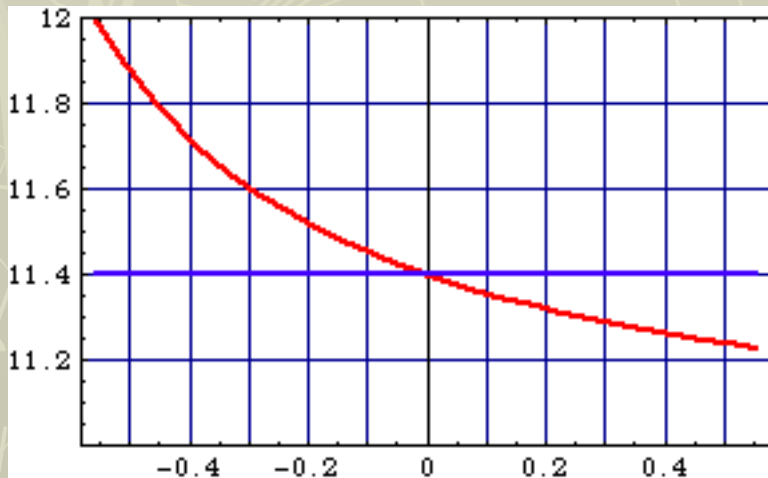
$\beta_V$  (m) vs. period length (m) for  $\delta = -0.6$  to  $+0.6$  in steps of 0.1



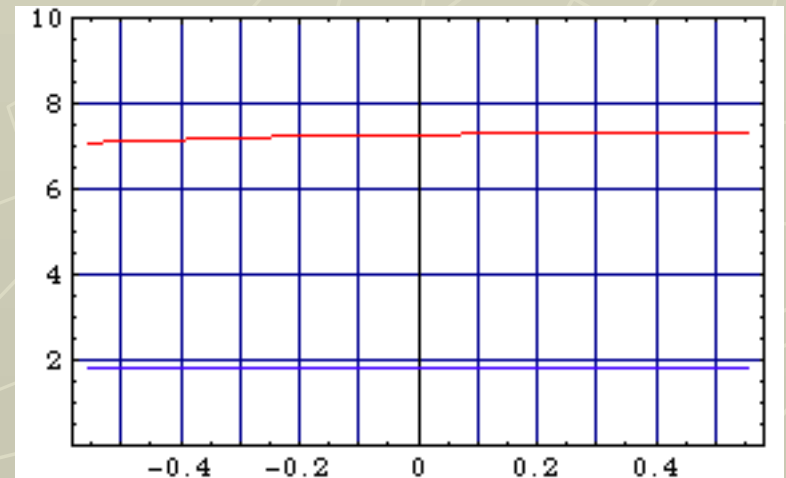
# Adjusted Field Profile (1 of 2)



Tune variation vs.  $\delta$   $\nu_H$  (Red)  $\nu_V$  (Blue)

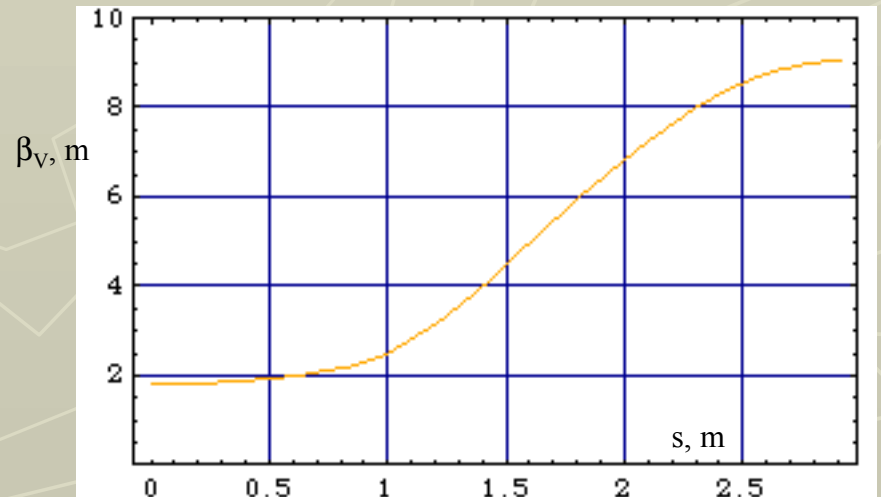
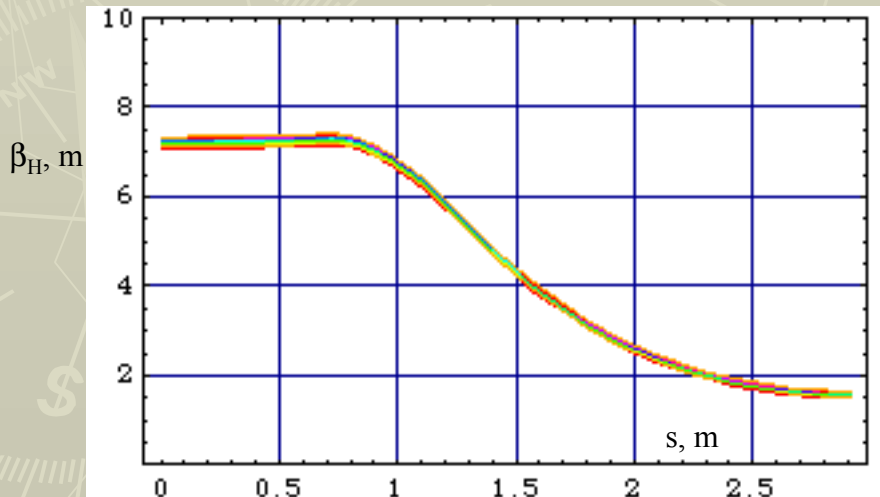
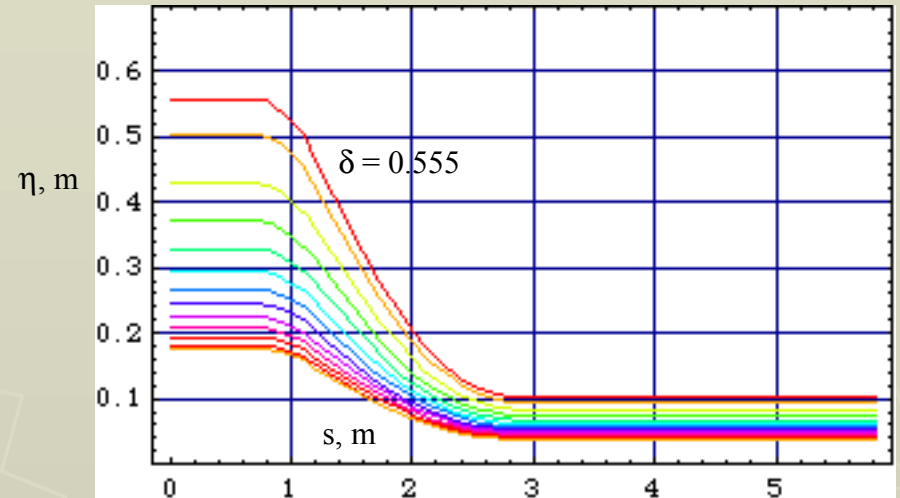
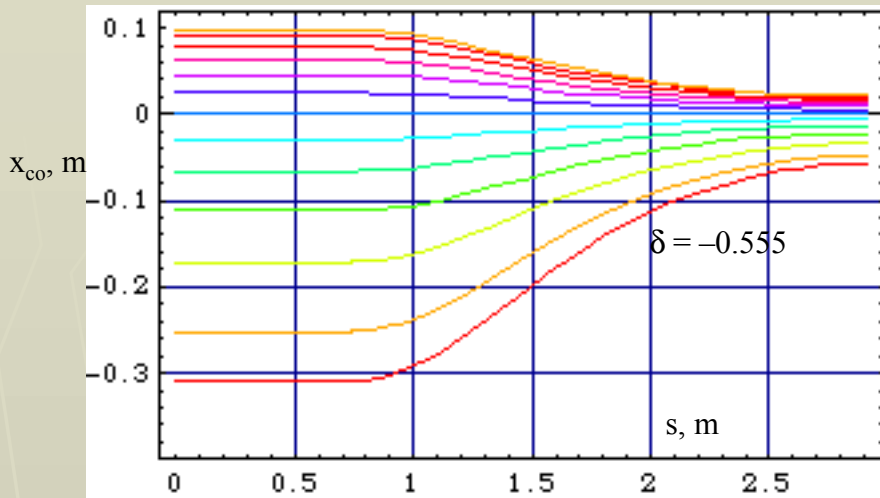
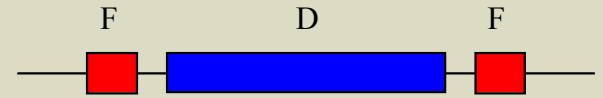


$\beta_H$  (Red) and  $\beta_V$  (Blue) in m vs.  $\delta$  at the beginning of a period (S)

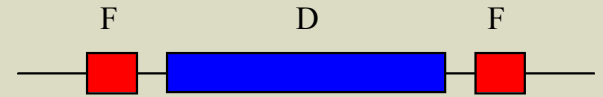




# Adjusted Field Profile (2 of 2)



# Sandro's Recipe (1 of 3)



- Hamiltonian in Curvilinear Coordinate System  $(x, s, y)$

$$H = -q A_s / c - (1 + h x) [ p^2 - (p_x - q A_x / c)^2 - (p_y - q A_y / c)^2 ]^{1/2}$$

- (1) Expand square root

$$H = -q A_s / c - (1 + h x) p + (p_x - q A_x / c)^2 / 2 p + (p_y - q A_y / c)^2 / 2 p$$

- (2) Drop higher order terms like  $hx(p_x - qA_x/c)^2$  and  $hx(p_y - qA_y/c)^2$
- (3) Assume that the magnetic field is given solely by the longitudinal component  $A_s$  of the vector potential, whereas identically  $A_x = A_y = 0$

$$x'' = (q / pc) \partial A_s / \partial x + h \quad \leftarrow \text{Curvature Function } h = h(s)$$

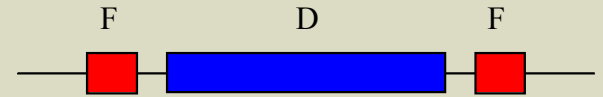
$$y'' = (q / pc) \partial A_s / \partial y$$

- Magnetic Field Components

$$(1 + hx) B_y = \partial A_s / \partial x$$

$$(1 + hx) B_x = - \partial A_s / \partial y$$

# Sandro's Recipe (2 of 3)



- Equations of motion are now

$$x'' = (q / pc) B_y (1 + hx) + h$$

$$y'' = - (q / pc) B_x (1 + hx)$$

- Quite generally  $B(z) = B_0 + G(z) z$

$$B = B_y + i B_x \quad z = x + i y$$

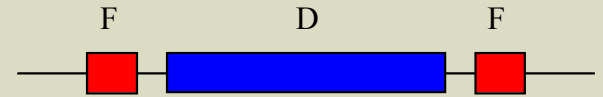
$$B_x = \text{Imaginary } \{B(z)\} \quad \text{and} \quad B_y = \text{Real } \{B(z)\}$$

- Motion on the  $y = 0$  mid-plane

$$x'' = (q / pc) [B_0 + G(x) x] (1 + hx) + h$$

- Lorenz Condition  $(q B_0 / p_0 c) = - h$  with  $p = p_0 (1 + \delta)$

# Sandro's Recipe (3 of 3)



- (4) Neglect the higher order term  $(q / pc) G h x^2$
- Introduce the *field index*  $n(x) = G(x) / h B_0$

$$\begin{aligned} x'' + h^2 (1 + n) x / (1 + \delta) &= h \delta / (1 + \delta) \\ y'' - h^2 n y / (1 + \delta) &= 0 \end{aligned}$$

- Consider the general case where the field index is a nonlinear function of both  $x$  and  $s$ , namely  $n = n(x, s)$ . At any location  $s$ , for each momentum value  $\delta$  there is one unique solution  $x = x(\delta, s)$ , and by *inversion*  $\delta$  is a function of  $x$  and  $s$ , namely  $\delta = \delta(x, s)$ . We pose the following problem: Determine the field distribution, namely  $n = n(x, s)$ , that compensates the momentum dependence of  $(1 + \delta)$  at the denominator:

$$n(x, s) = n_0 [1 + \delta(x, s)] \quad \text{-->} \quad G(x, s) = G_0 [1 + \delta(x, s)]$$

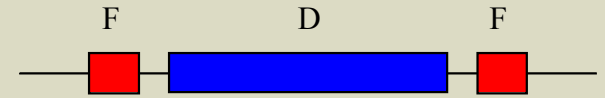
where  $n_0$  is related to the gradient  $G_0 = n_0 h B_0$  on the reference trajectory.

- Then the equations of motion reduce to

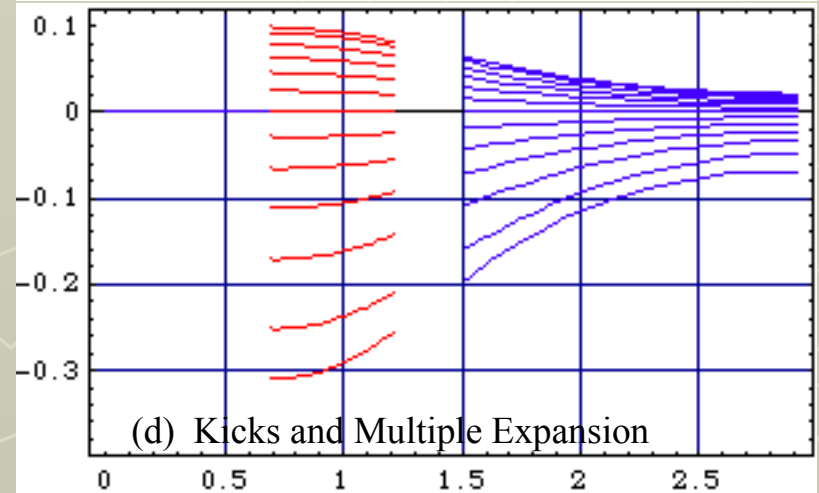
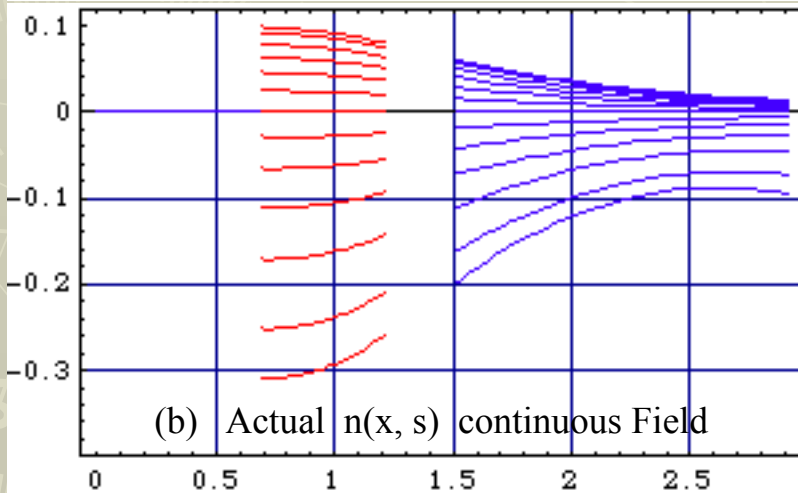
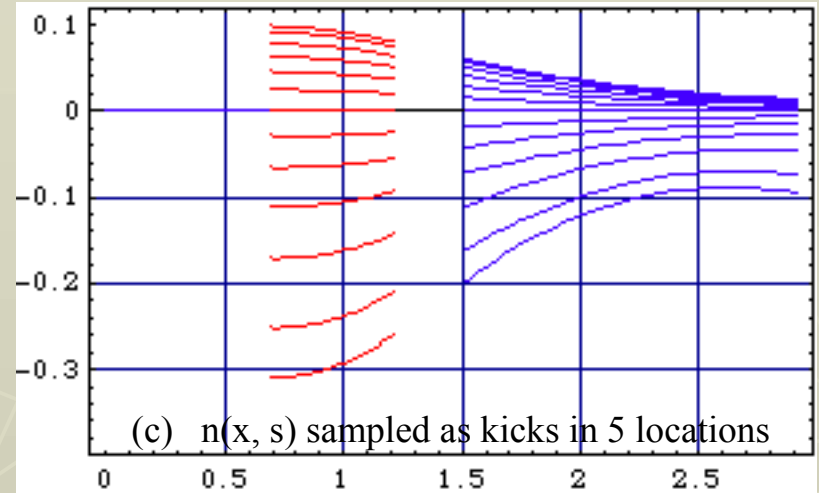
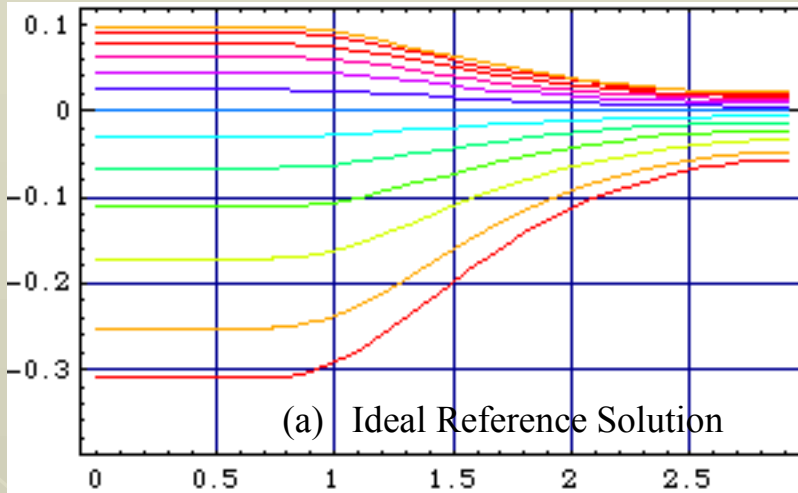
$$\begin{aligned} x'' + h^2 x / (1 + \delta) + h^2 n_0 x &= h \delta / (1 + \delta) \quad \text{-->} \quad x = x(\delta, s) \quad \text{-->} \quad \delta = \delta(x, s) \\ y'' - h^2 n_0 y &= 0 \end{aligned}$$

- **WARNING:** (5) A variation with  $s$  of the guiding field introduce a *solenoid component* that must be evaluated and taken into account in the particle dynamics.

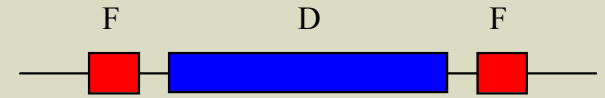
# Proof of Sandro's Recipe (1 of 2)



By Direct **Integration** of  $x'' + h^2 [1 + n(x, s)] x / (1 + \delta) = h \delta / (1 + \delta)$



# Proof of Sandro's Recipe (2 of 2)

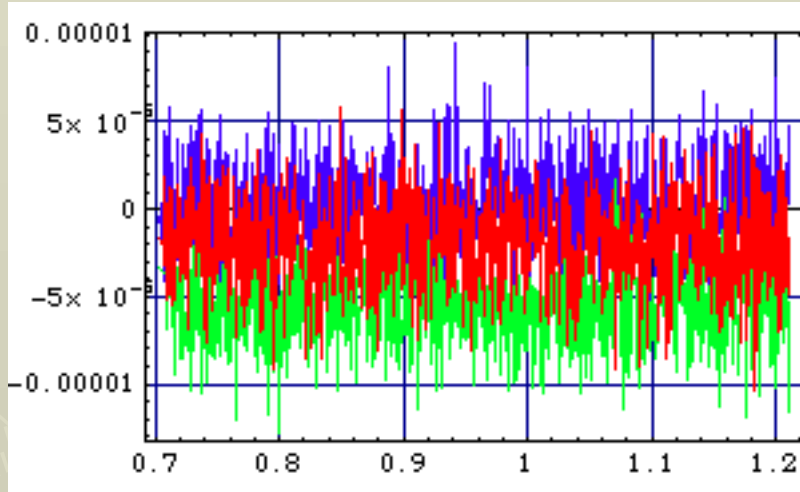


By Direct **Substitution** of  $x_{co}(\delta, s)$  in  $x'' + h^2 [1 + n(x, s)] x / (1 + \delta) = h \delta / (1 + \delta)$

F - Sector

$\delta =$

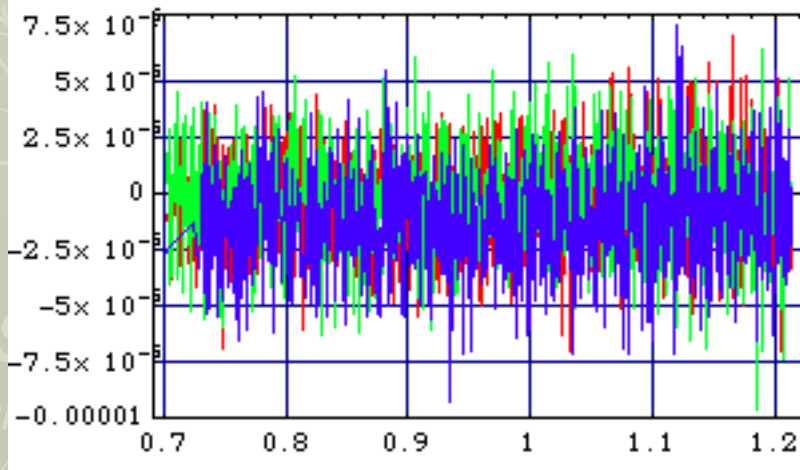
D - Sector



-0.55

-0.4

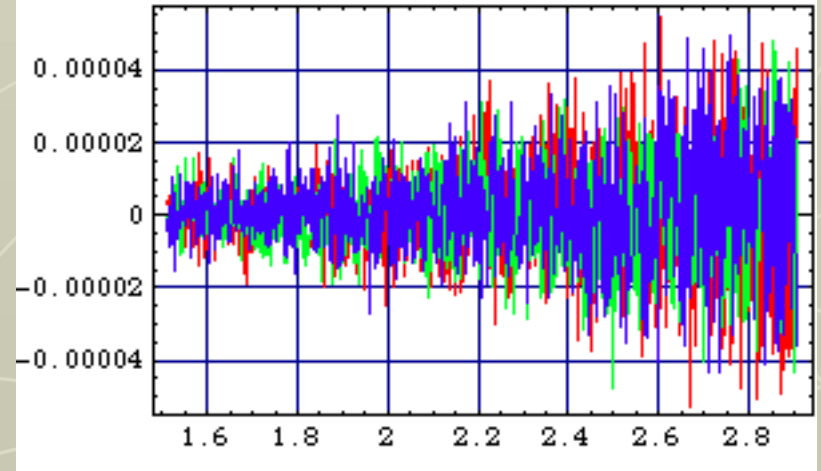
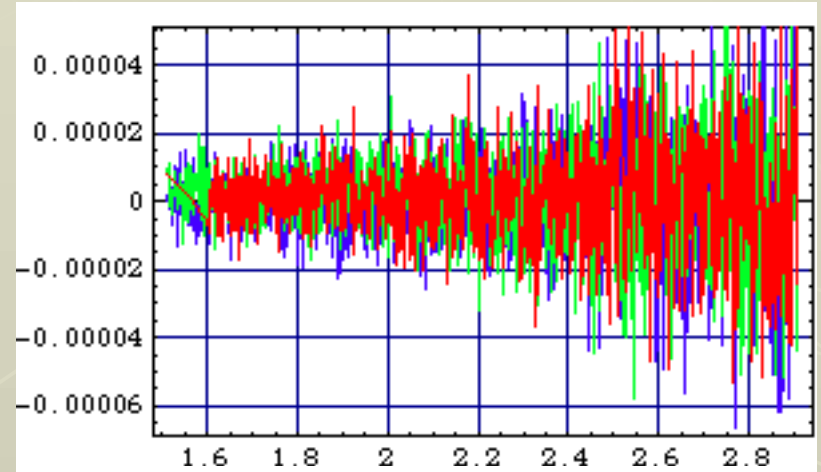
-0.2



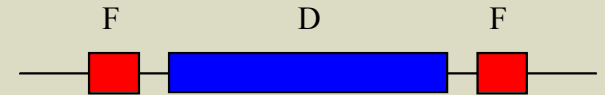
0.2

0.4

0.55



# Alternative Solutions



Other solutions are also possible. For instance, in order to flatten entirely the  $\delta$ -dependence of the horizontal betatron tune, one can set the field profile so that

$$1 + n(x, s) = (1 + n_0) [1 + \delta(x, s)]$$

leading to the equations of motion

$$\begin{aligned} x'' + h^2 (1 + n_0) x &= h \delta / (1 + \delta) \\ y'' - h^2 [\delta / (1 + \delta) + n_0] y &= 0 \end{aligned}$$

There is now a (reduced)  $\delta$ -dependence of the vertical betatron tune.

There is, of course, still dispersion on the horizontal plane.

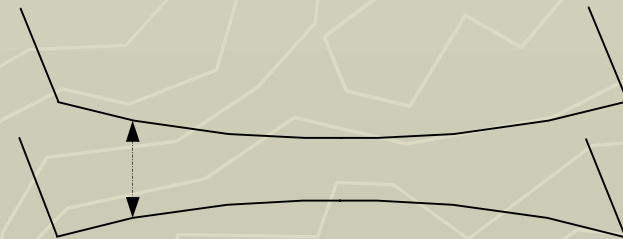
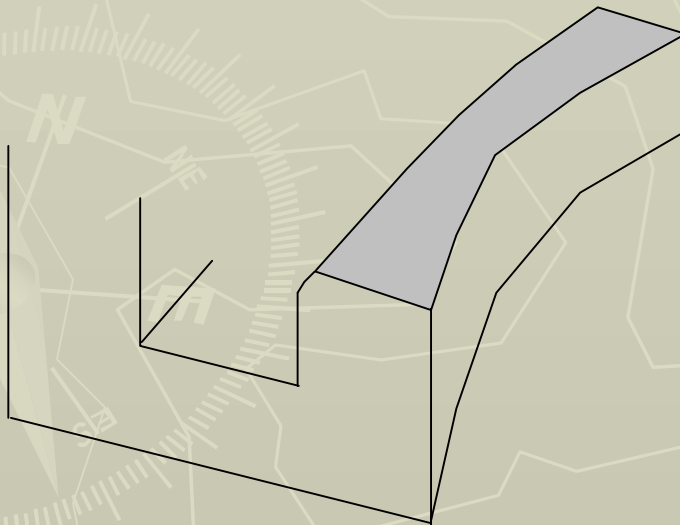
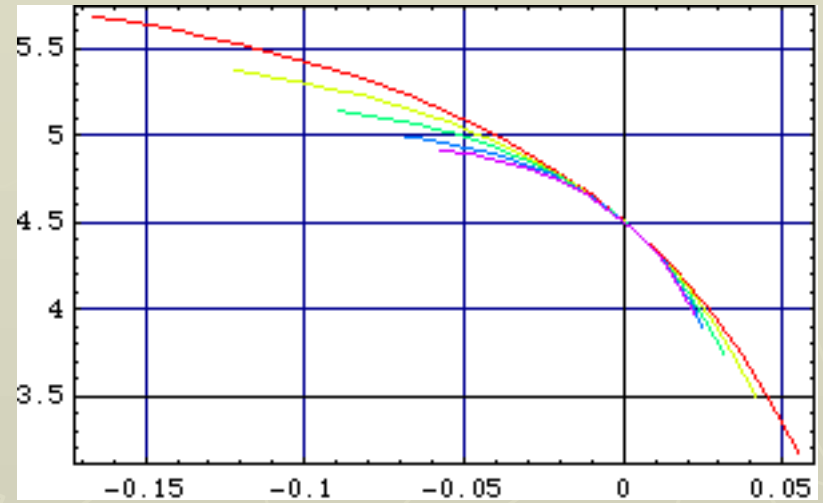
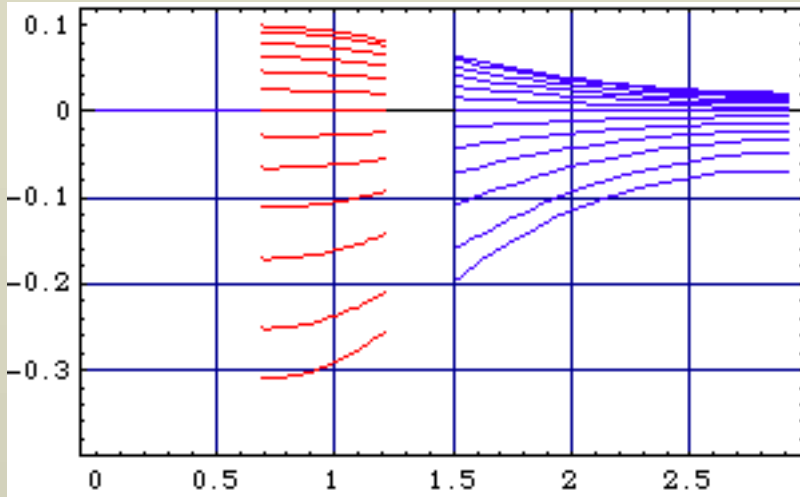
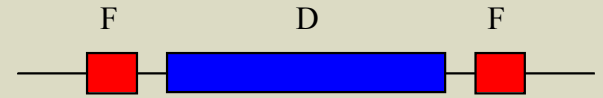
The field profile on the  $y = 0$  mid-plane associated to this solution is given by

$$B(x, s) = B_0 + G_0 [1 + \delta(x, s) (1 + 1/n_0)]$$

that, as long  $n_0 \gg 1$ , is only slightly different from the previously derived field profile.

If desired, always in the search for an optimum cancellation of the chromatic and dispersive behavior, intermediate solutions can be found.

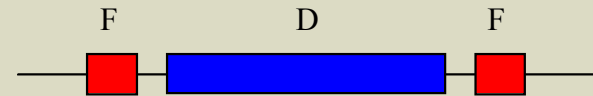
# Magnet Design



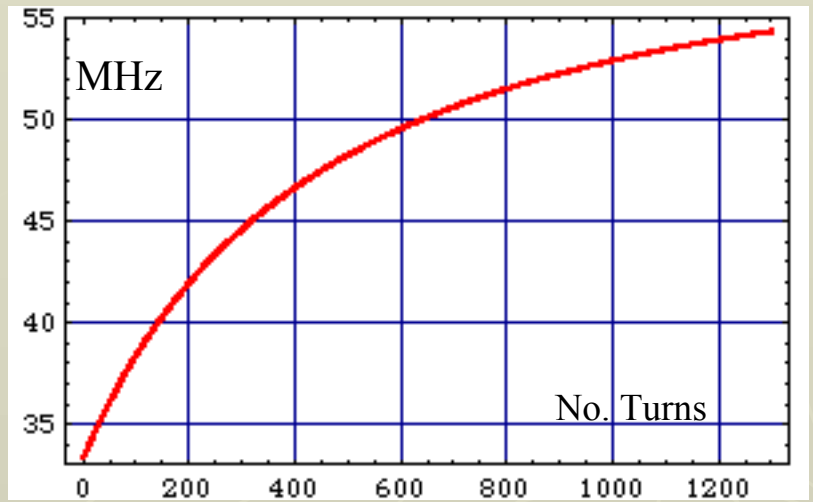
Minimum Gap      10 cm  
 Maximum Width    40 cm



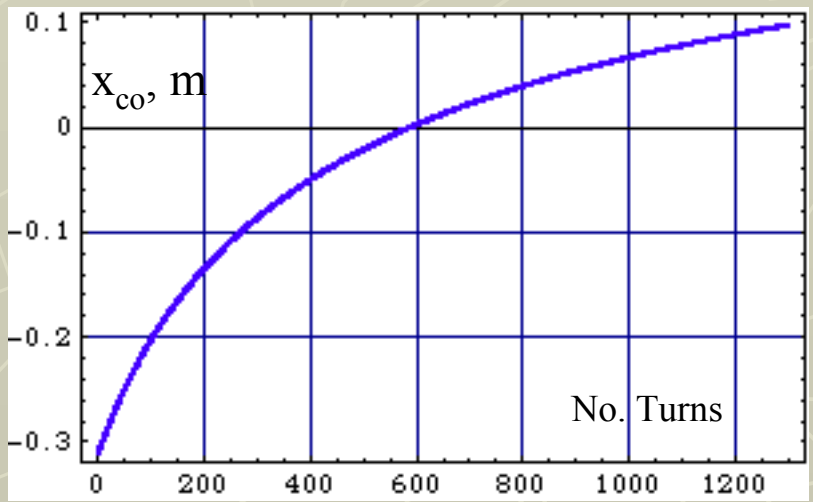
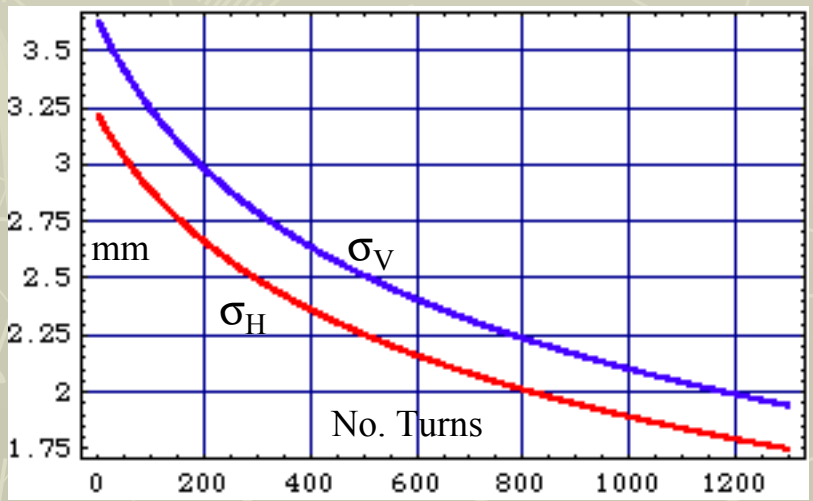
# Acceleration: Frequency Modulation



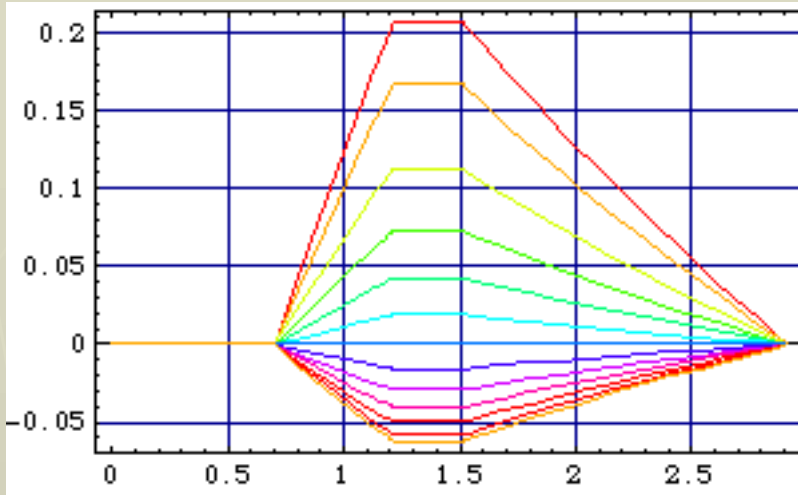
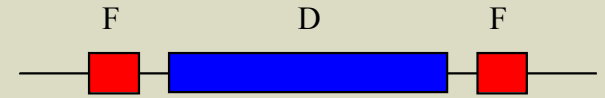
Energy Range	0.2 - 1.5 GeV
Energy Gain	1 MeV/turn
No. of Revolutions	1,300
Acceleration Period	1.31 ms
Harmonic No.	48
RF Swing	33 - 55 MHz
No. of Protons	$2.5 \times 10^{13}$ ppp
Beam (RF) Peak Power	2.75 - 4.5 MW
RF @ Inj / 201.25 MHz	1/6



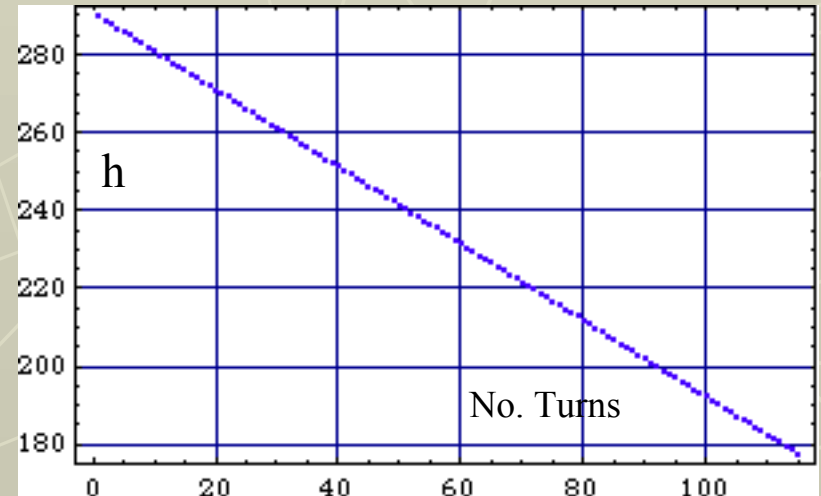
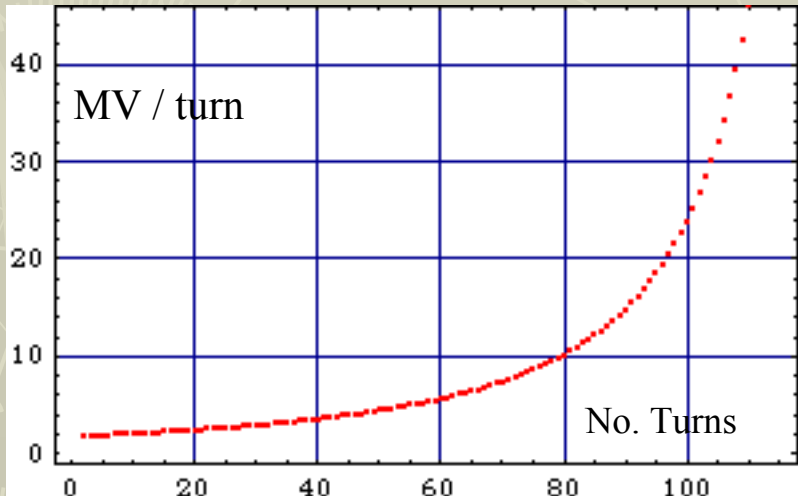
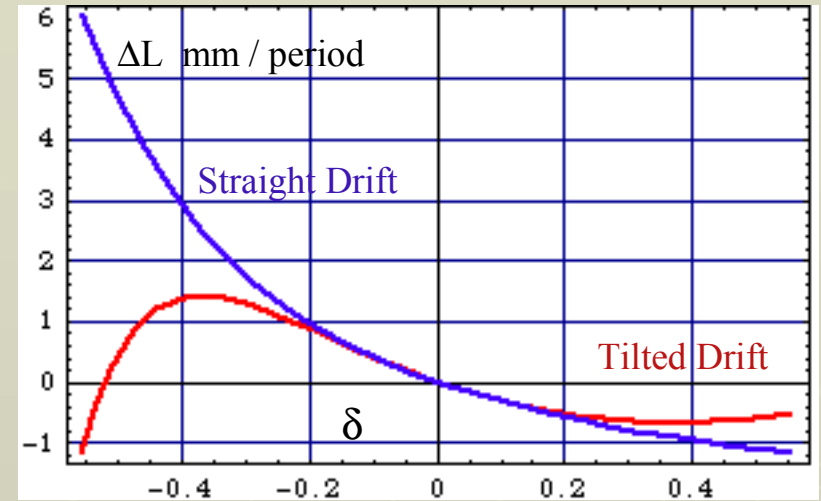
Single-Gap Cavity: 1.2 m long 50-100 kV  
 W: 40 cm H: 10 cm



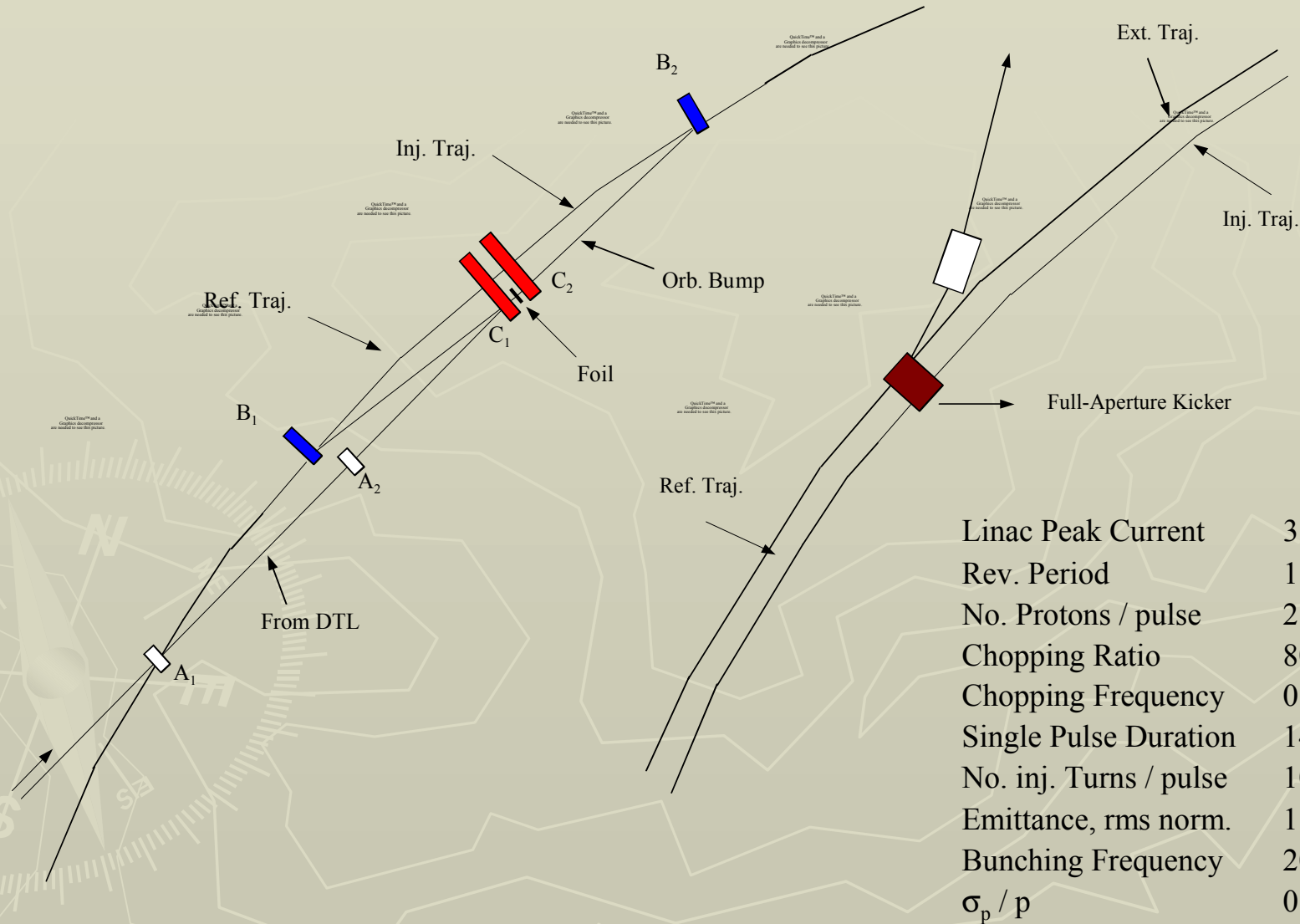
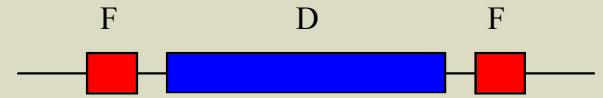
# Acceleration: Voltage Modulation



$$eV = E_0 \beta^2 \gamma / n (1 - \alpha_p \gamma)$$

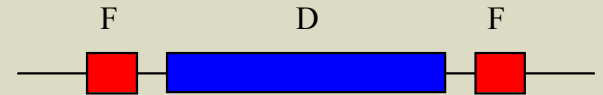


# Injection (H<sup>-</sup>) & Extraction



Linac Peak Current	35 mA
Rev. Period	1.44 $\mu$ s
No. Protons / pulse	$2.5 \times 10^{13}$
Chopping Ratio	80 %
Chopping Frequency	0.694 MHz
Single Pulse Duration	144 $\mu$ s
No. inj. Turns / pulse	100
Emittance, rms norm.	1 $\pi$ mm mrad
Bunching Frequency	201.25 MHz
$\sigma_p / p$	0.1 %

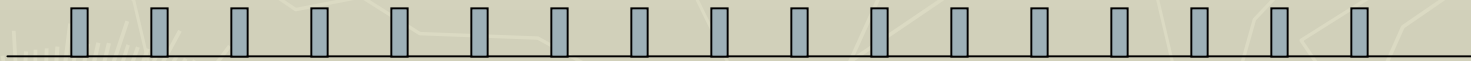
# Conclusions



The **1.5-GeV FFAG** is an attractive alternative to the **1.2-GeV SCL** as the new injector for the **AGS Upgrade** program. The merits are:

- More familiar and conventional technology
- Less expensive
- Possibility of acceleration of Heavy Ions

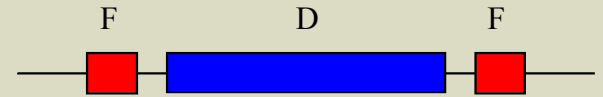
More work has clearly to be done before it is considered as a substitute to the SCL.



By extrapolation, it is also a continuous high power **Proton Driver** for a variety of applications:

Final Energy	1.5 GeV
Repetition Rate	670 Hz
Protons / Pulse	$2.5 \times 10^{13}$
Average Beam Power	<b>4.0 MWatt</b>

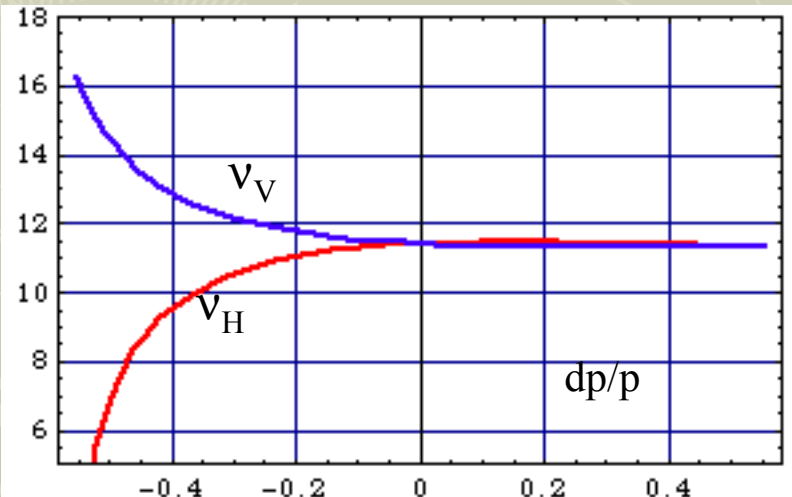
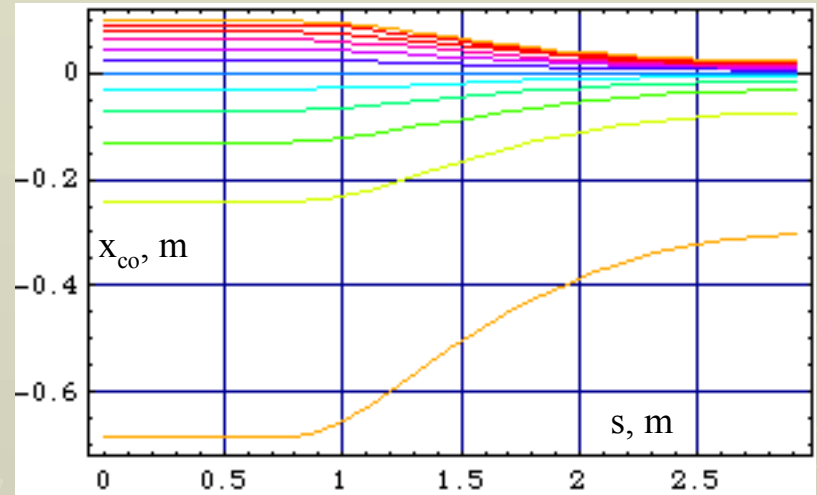
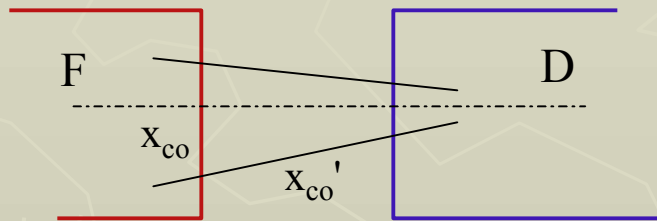
# Edge Effect



$$\Delta y' = - \frac{h_i B_i(x_{co})}{B_i (1 + \delta)} (\tan x_{co}') y$$

Vertical == Focusing

Horizontal == Defocusing



- Edge Effect is important and it cannot be neglected. In the case of the FDF-triplet arrangement there is a problem for large negative values of  $\delta$ .

- If we accept the *recipe* for the **Adjusted Field Profile**, we could also apply it to the DFD-triplet arrangement. Maybe in this case the problem will shift to large positive values of  $\delta$  (let us check!...)

- I am aware that the DFD-triplet is desirable for the accommodation of injection/extraction components and RF cavities, but it could make the central magnet wider.

# DFD - Triplet

