Cost Optimization of FFAGs

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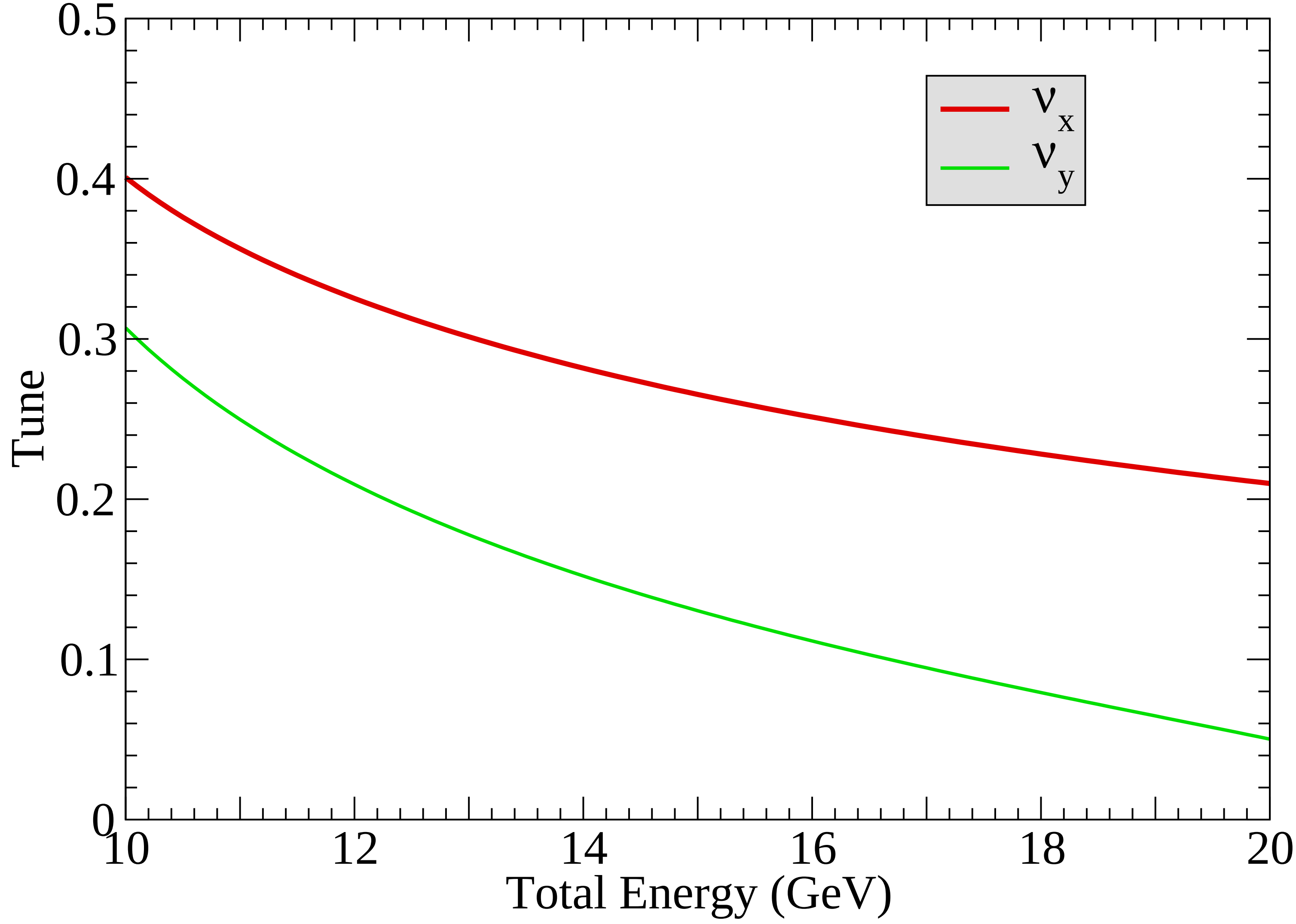
Introduction

- Want a method for producing the "best" lattice in some sense
- Need to have a semi-automated design method to accomplish this
- Apply standard optimization techniques to automated design method
- Outline
  - Describe how I design and optimize FFAG lattices
  - Give optimized lattices, showing historical progress
    - Analyze reasons behind improvements

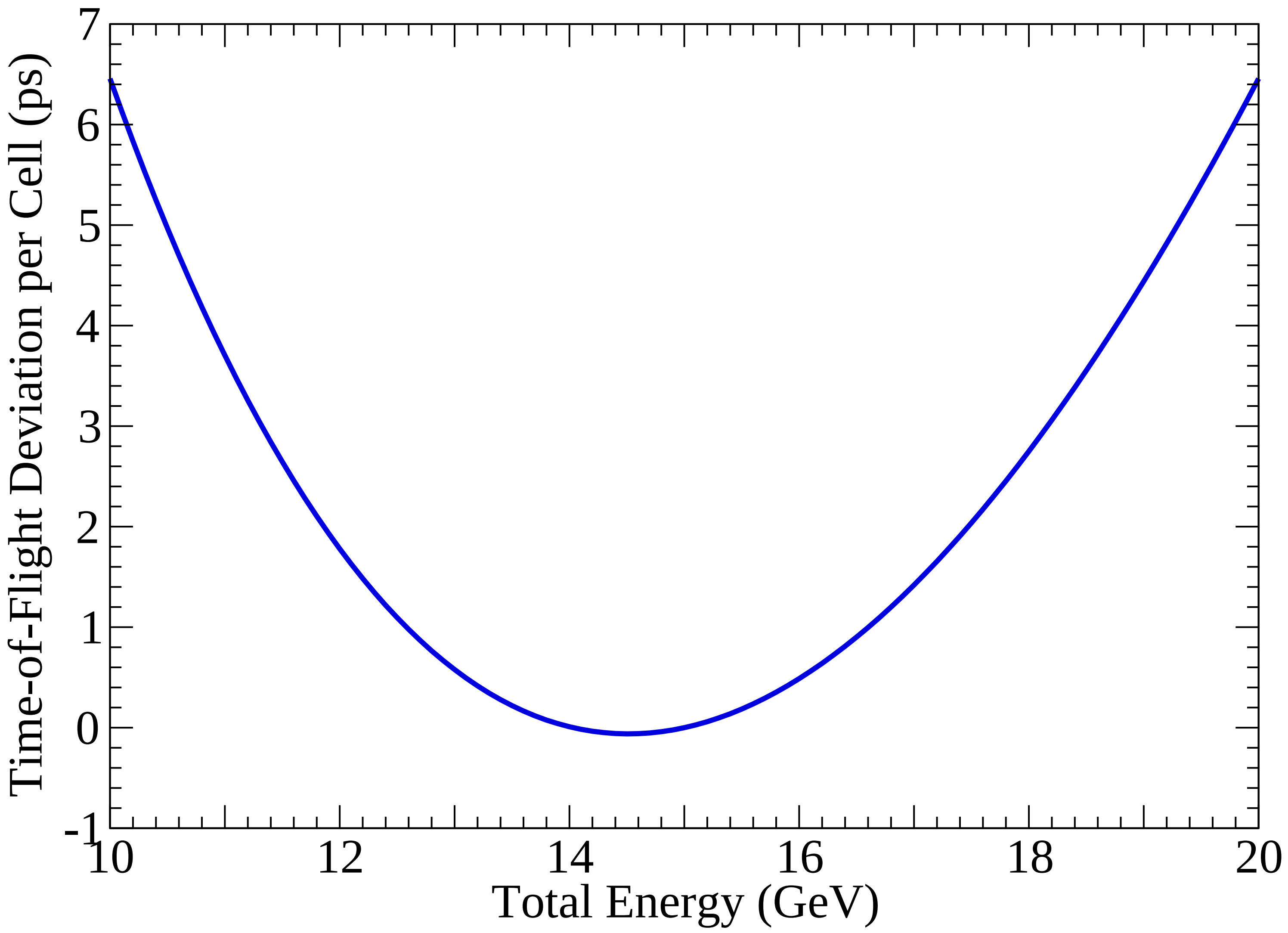
How to Analyze an FFAG Lattice

- Compute closed orbit as a function of energy
  - Closed orbit, time-of-flight variation with energy
  - Midplane symmetry: closed orbit stays in plane
- Compute the linear map about that closed orbit
  - Betatron functions, tunes
  - Midplane symmetry: no coupling
- Integrate through the magnet: do not use a formula!
  - Closed orbit is not parallel to the reference orbit
- Most simple linear FFAG lattices
  - Tune monotonically decreasing with energy
  - Time-of-flight variation with energy well approximated by a quadratic function

Tune Variation with Energy



Time-of-Flight vs. Energy



Determining Pipe Size

- Want round pipe that encloses the beam ellipse at every energy
  - Beam ellipse centered on closed orbit, which lies in midplane
  - Beta functions and emittance determine semi-axes of ellipse
- Determine circle inside which midplane-centered ellipse is inscribed
  - Ellipse k, center at $(c_k, 0)$, semi-axes a_k (horizontal) and b_k
  - Circle centered at $(z, 0)$, determine radius r so that circle is outside ellipse but just touching:

$$r_k^2(z) = \begin{cases} (c_k + a_k - z)^2 & z \leqslant z_{k-} \\ b_k^2 \left[\dfrac{(c_k - z)^2}{b_k^2 - a_k^2} + 1 \right] & z_{k-} < z < z_{k+} \\ (c_k - a_k - z)^2 & z_{k+} \leqslant z \end{cases}$$

$$z_{k-} = \min\left\{ c_k, c_k - \frac{b_k^2 - a_k^2}{a_k} \right\} \qquad z_{k+} = \min\left\{ c_k, c_k + \frac{b_k^2 - a_k^2}{a_k} \right\}$$

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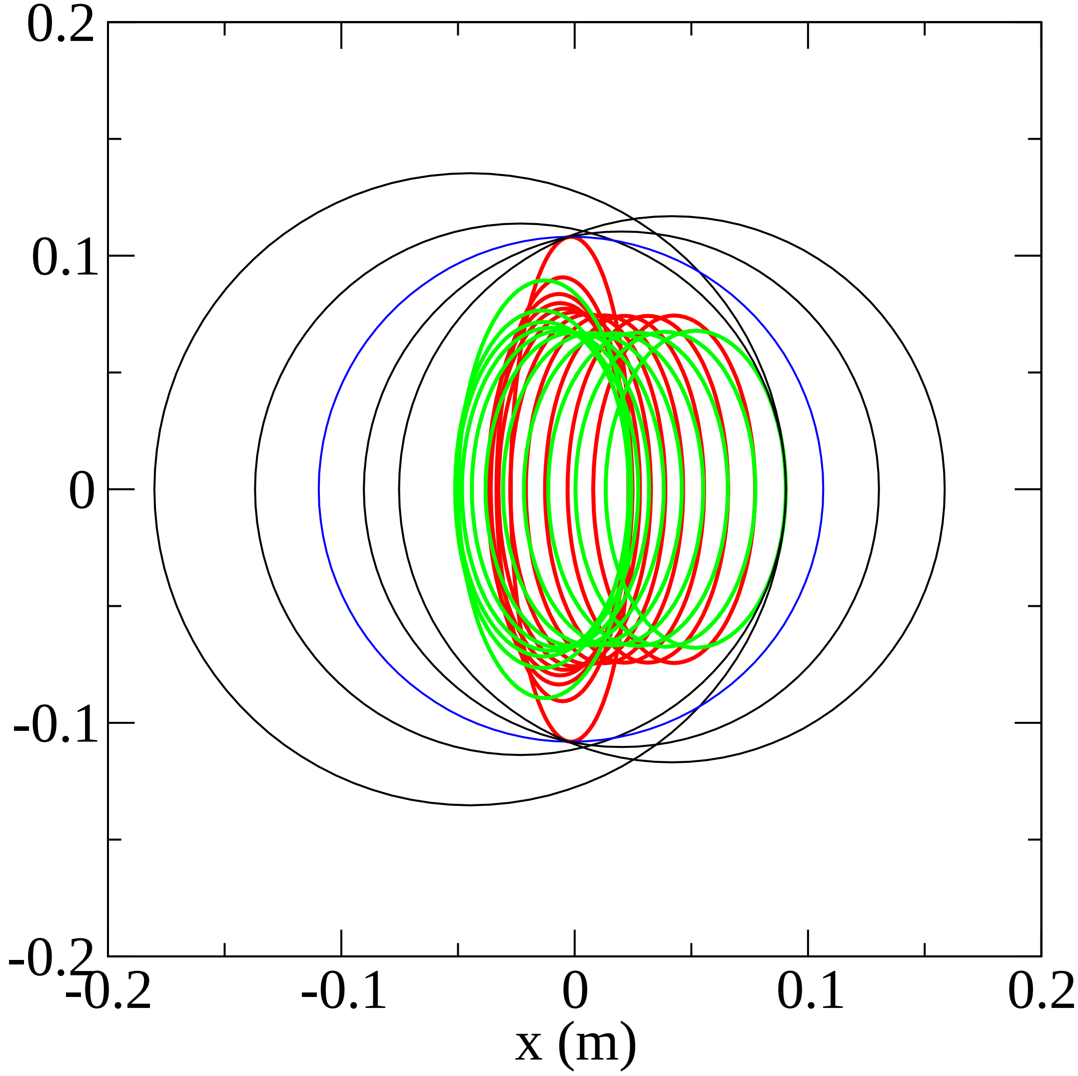
Determining Pipe Size (cont.)

Muon Collaboration

- Find circle enclosing all ellipses
  - For each z, find $R(z) = \max_k r_k(z)$
    - ⋆ For each k, keep track of intervals over which r_k is *not* largest
      - ➢ Involves intersection of quadratic functions: easy
    - ⋆ Throw out ellipse k when it is not largest for all z
      - ➢ Most ellipses thrown out quickly
      - ➢ Algorithm becomes very fast, even for many ellipses
  - Determine optimum z
    - ⋆ Take smallest $R(z)$; easy since piecewise quadratic
    - ⋆ Optimize cost formula: in many cases, this is smallest $R(z)$

Circles Enclosing Ellipses



Edge Effects

- Want effect of magnet ends without having to know specific shape of end
- Can compute this effect exactly under the following assumptions:
  - First order in the strength of the body field
  - Limit as the length of the end goes to zero
- With midplane symmetry and only wanting linear effects, the answer is

$$\Delta p_y = -\frac{q y p_x}{p_s} \Delta B_{y0}(x)$$

  - Coordinate system defined relative to pole face (x parallel to pole face)
  - $\Delta B_{y0}(x)$ is change in midplane field (positive when entering magnet with positive B_y
- Small effect for higher-energy muon machines: lots of cells

Modified Palmer Cost Model

$$B_{i\pm} = B_0 \pm B_1 f_R R$$
$$B_{o\pm} = B_0 \pm B_1 R_{o\pm}$$
$$C_\pm = C_{m0} B_{e\pm}^{1.5} R_{o\pm}(L + f_E f_R R)$$
$$f_Q = \frac{|B_{e+} + B_{e-}|}{|B_{e+}| + |B_{e-}|} k_D + \frac{|B_{e+} - B_{e-}|}{|B_{e+}| + |B_{e-}|} k_Q$$
$$C_{\text{mag}} = (C_+ + C_-)(n_0/n)^{1/3} f_A f_Q / 2$$

$$R_{o\pm} = f_R R + t_C |B_{i\pm}|$$
$$B_{e\pm} = \begin{cases} B_{i\pm} & |B_{i\pm}| \geqslant |B_{o\pm}| \\ B_{o\pm} & |B_{i\pm}| < |B_{o\pm}| \end{cases}$$
$$C_{\text{rf}} = k_C V G_0 / G + k_P V G / G_0$$
$$C_{\text{lin}} = C_L L_R$$

- Costs C_{mag} (magnets), C_{rf} (RF), and C_{lin} (linear)
- n magnets, magnet radius is $f_R R$, magnet length is L, central field is B_0, gradient is B_1
- Total installed voltage is V, RF gradient is G, ring length is L_R
- PB is our cost unit, the “Palmer Buck”

f_R	1.3	t_C	2 mm	C_{m0}	22.5 mPB/T$^{1.5}$/m^2	f_E	20
k_D	1	k_Q	1.5	n_0	300	f_A	1.5
k_C	30 PB/GV	k_P	26.8 PB/GV	G_0	16 MV/m	C_L	25 mPB/m

Cost Behavior

- Short magnet, depends on R^2
  - Relatively independent of L: making magnet longer very cost effective
    - ⋆ Beware: R increases weakly with increasing L
    - ⋆ Note large f_E
- Longer magnet, depends linearly on R
  - May not be cost-effective to lengthen
- Optimum tends to occur when $L \approx f_E f_R R$
- Superconducting RF: optimum gradient is generally the highest you can achieve

Longitudinal Dynamics

- Normalized longitudinal acceptance in scaled variables ($E/\Delta E$ and ωt) depends on two dimensionless parameters

$$w = \frac{V}{\omega \Delta T \Delta E} \qquad z = \frac{T_0}{\Delta T}$$

  - $\Delta E = E_{\text{max}} - E_{\text{min}}$
  - ω is the angular RF frequency
  - V is voltage per cell/turn
  - ΔT is the height of the time-of-flight parabola for one cell/turn (difference between maximum and minimum times-of-flight)
  - At the energy with the shortest time-of-flight, if the particle arrives at a cavity with a phase ϕ, it arrives at the cavity in the next cell with a phase $\phi - \omega T_0$.

Longitudinal Dynamics (cont.)

- Larger w, larger longitudinal acceptance
  - $w > 1/24$ in any case
  - Assumes a perfectly parabolic time-of-flight dependence
- Note required w increases with decreasing energy
  - Normalized longitudinal emittance required is same for all energies
  - ΔE smaller for lower energy machines
  - Thus, scaled emittance is larger
- $\Delta T \propto (\Delta E)^2$ for parabolic variation
  - For given w, $V \propto (\Delta E)^3$
- Lowering ΔT increases w and/or decreases required V
- Lower ω gives lower V/larger w
- In some regimes, scaled emittance proportional to w^2

Longitudinal Dynamics: Design Tradeoffs

- ΔT per cell proportional to $1/n^2$, n is the number of cells
- Thus, for given w, voltage per turn proportional to $1/n$
- Magnet costs proportional to n if individual magnet costs independent of n
  - Do even better: higher n, smaller closed orbit swing, small aperture
  - Per-magnet costs go down with more magnets
- ΔT proportional to cell length for a given cell type

Basic Non-Scaling FFAG Design Principles

- Items that can vary
  - Integrated dipole fields: (number of cells, ΔT, center parabola)
  - Integrated gradients: (tunes, beta functions, weak ΔT dependence)
    - Only horizontal lattice affects ΔT
  - Magnet lengths: (pole tip fields, weak cell length dependence)
  - 6 parameters total
- Don't over-focus: low energy tune below 0.5.
- ΔT determined by
  - Longitudinal acceptance (w)
  - Available voltage per cell

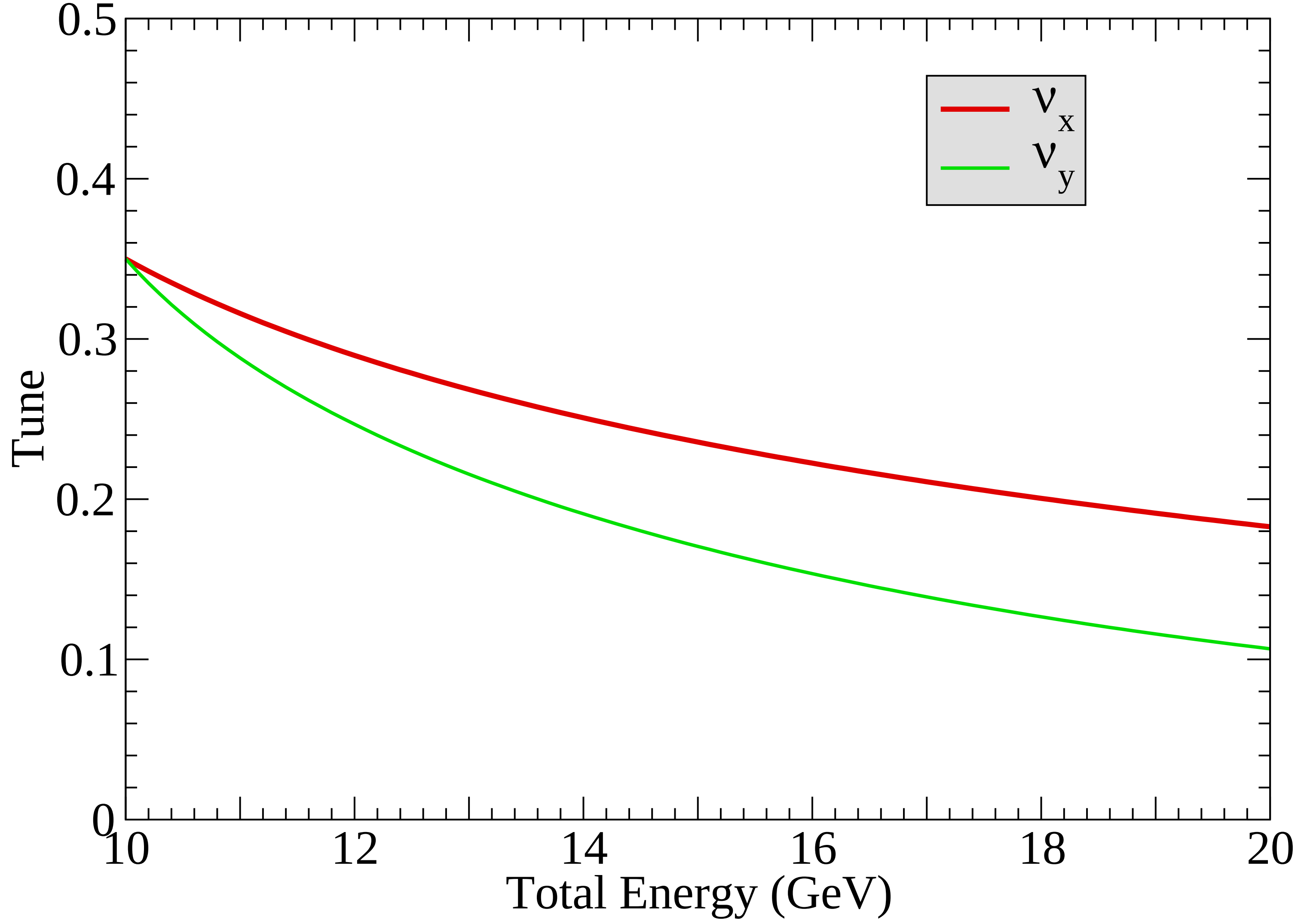
Muon Lattice Input Parameters

- 201.25 MHz superconducting RF
  - 2 m drift needed for RF cavities: stray field from magnets
  - 7.5 MV per cell (Cornell achieved)
- 0.5 m between magnets
- Normalized acceptance: 30 mm (acceptance times beta function is square of half-aperture)
- Factor of 2 in energy
- w is 1/12 for 10–20 GeV, 1/8 for 5–10 GeV, 1/6 for 2.5–5 GeV

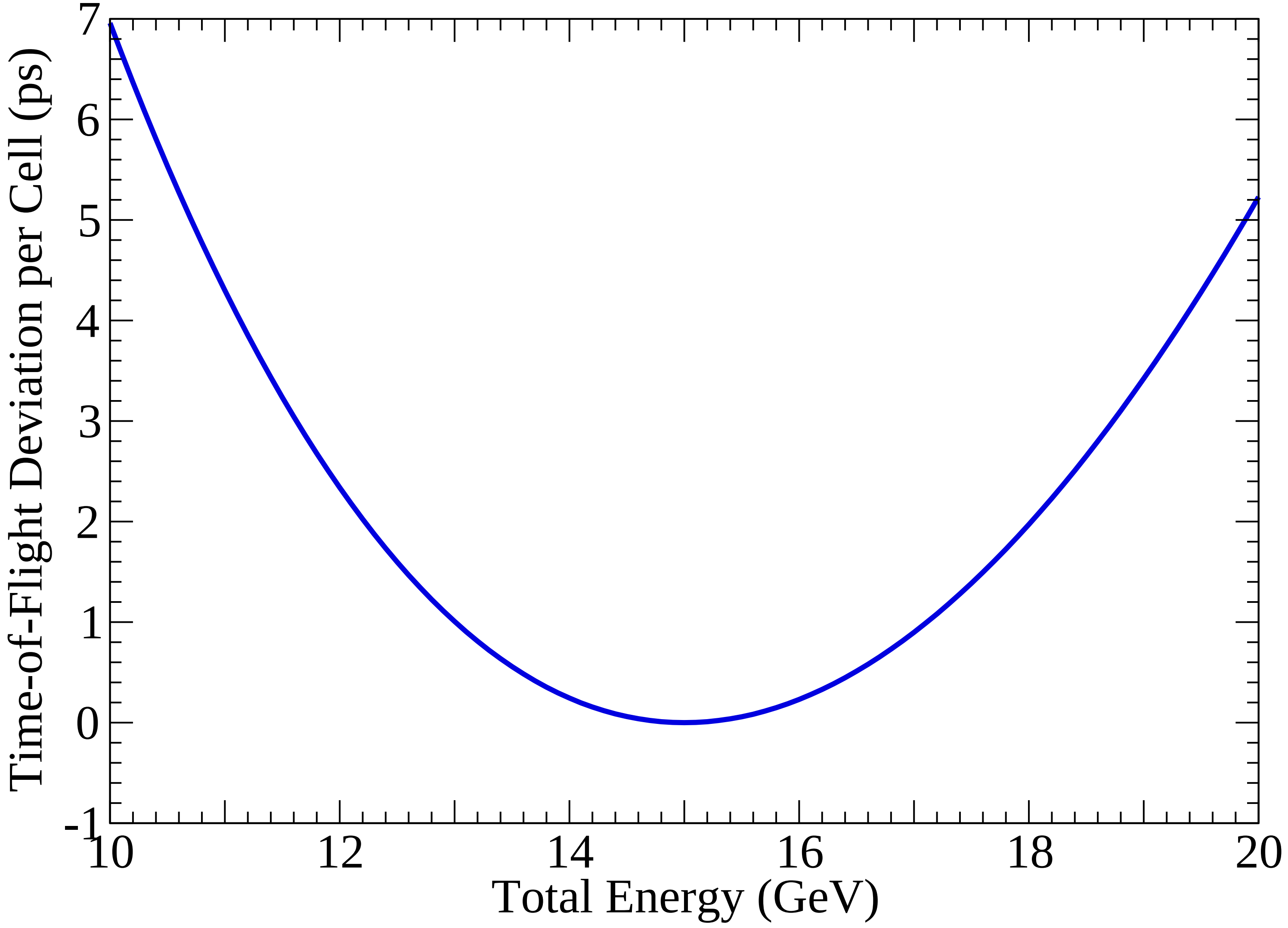
Fixed Pole Tip Fields

- Require 7 T pole tip fields
  - Achievable but not too crazy
- Fix low-energy tunes at 0.35
- Fix derivative of time-of-flight to be zero at central energy
- System completely determined (remaining parameter is w)
  - w determined using low-energy time-of-flight
    - ⋆ Higher than high-energy time-of-flight
    - ⋆ Asymmetric because not perfectly quadratic
- Show 10–20 GeV only
- Show different lattice types

Tune Variation with Energy



Time-of-Flight vs. Energy



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Fix Pole Tip Fields (cont.)

Muon Collaboration

Type	FDF	FD	FODO
Cells	93	101	113
D Length (cm)	128	101	81
D Radius (cm)	8.4	6.9	8.0
F Length (cm)	45	81	60
F Radius (cm)	9.5	12.4	14.2
RF Voltage (MV)	698	758	848
$c\Delta T$ (cm)	19.9	21.6	24.1
Circumference (m)	481	436	612
Magnet cost (PB)	76	69	90
RF cost (PB)	45	49	55
Linear cost (PB)	12	11	15
Total cost (PB)	134	129	161

- Triplet has shortest time-of-flight and lowest voltage requirement
- Doublet has lowest cost
  - Fewer magnets than triplet
- FODO does poorly
  - Relatively long cell

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Fix Tunes, Cost Optimized

Muon Collaboration

Type	FDF	FD	FODO
Cells	108	113	127
D Length (cm)	175	137	130
D Radius (cm)	10.2	8.7	9.7
D Pole Tip (T)	4.4	4.6	4.0
F Length (cm)	118	221	213
F Radius (cm)	11.9	13.8	15.5
F Pole Tip (T)	2.4	2.3	1.9
RF Voltage (MV)	811	849	950
$c\Delta T$ (cm)	23.1	24.1	27.1
Circumference (m)	768	688	941
Magnet cost (PB)	39	34	33
RF cost (PB)	53	55	62
Linear cost (PB)	19	17	24
Total cost (PB)	111	106	118

- Remove constraint on pole tip fields
- Find minimum cost varying magnet lengths
- Still 10–20 GeV, different lattices
- Pole tip fields went down
  - Magnets were too short (cost formula): little cost in lengthening
  - Lower fields in F quad: less bending?
- Longer magnets lengthened cell, made voltage slightly more
- Cost down significantly from higher pole tip fields
- FODO not so bad

Fix Tunes, Cost Optimized (cont.)

Min Energy (GeV)	2.5	5	10
Max Energy (GeV)	5	10	20
$V/\omega\Delta T\Delta E$	1/6	1/8	1/12
Cells	80	93	108
D Length (cm)	144	158	175
D Radius (cm)	19.0	13.8	10.2
D Pole Tip (T)	2.0	3.1	4.4
F Length (cm)	84	96	118
F Radius (cm)	18.2	14.4	11.9
F Pole Tip (T)	1.2	1.8	2.4
RF Voltage (MV)	604	695	811
$c\Delta T$ (cm)	34.3	26.4	23.1
$\Delta E/V$	4.1	7.2	12.3
Circumference (m)	493	603	768
Magnet cost (PB)	25	31	39
RF cost (PB)	39	45	53
Linear cost (PB)	12	15	19
Total cost (PB)	77	91	111
Cost per GeV (PB)	30.8	18.2	11.1

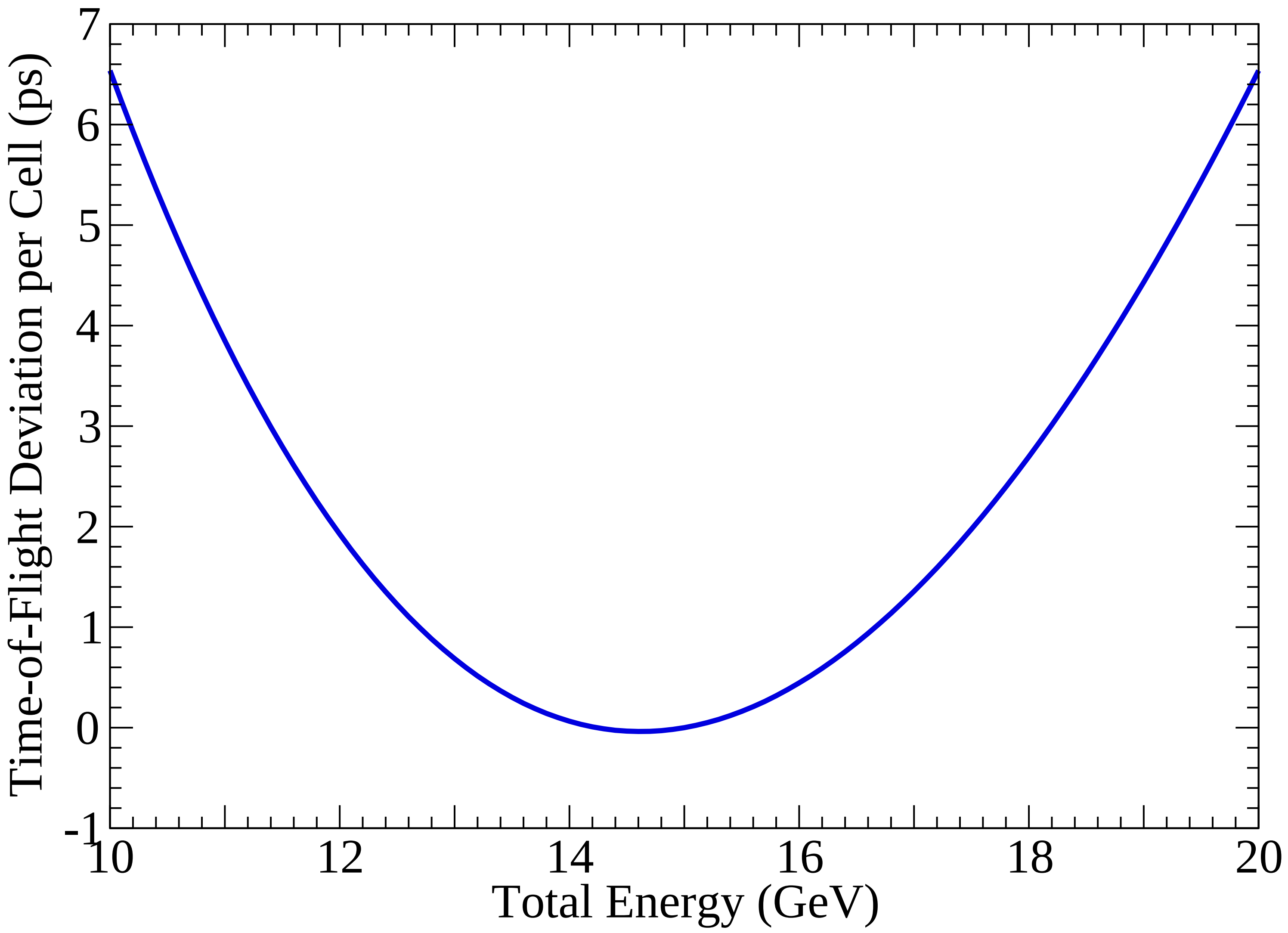
- Triplet lattices, different energies
- Cost per GeV increases substantially as energy lowers
  - Increasing magnet apertures
  - Increasing $V/\omega\Delta T\Delta E$
  - Number of cells decreases very slowly
- 2.5–5 GeV not cost-effective: only 4 “equivalent turns”

Different Tunes, Cost Optimized

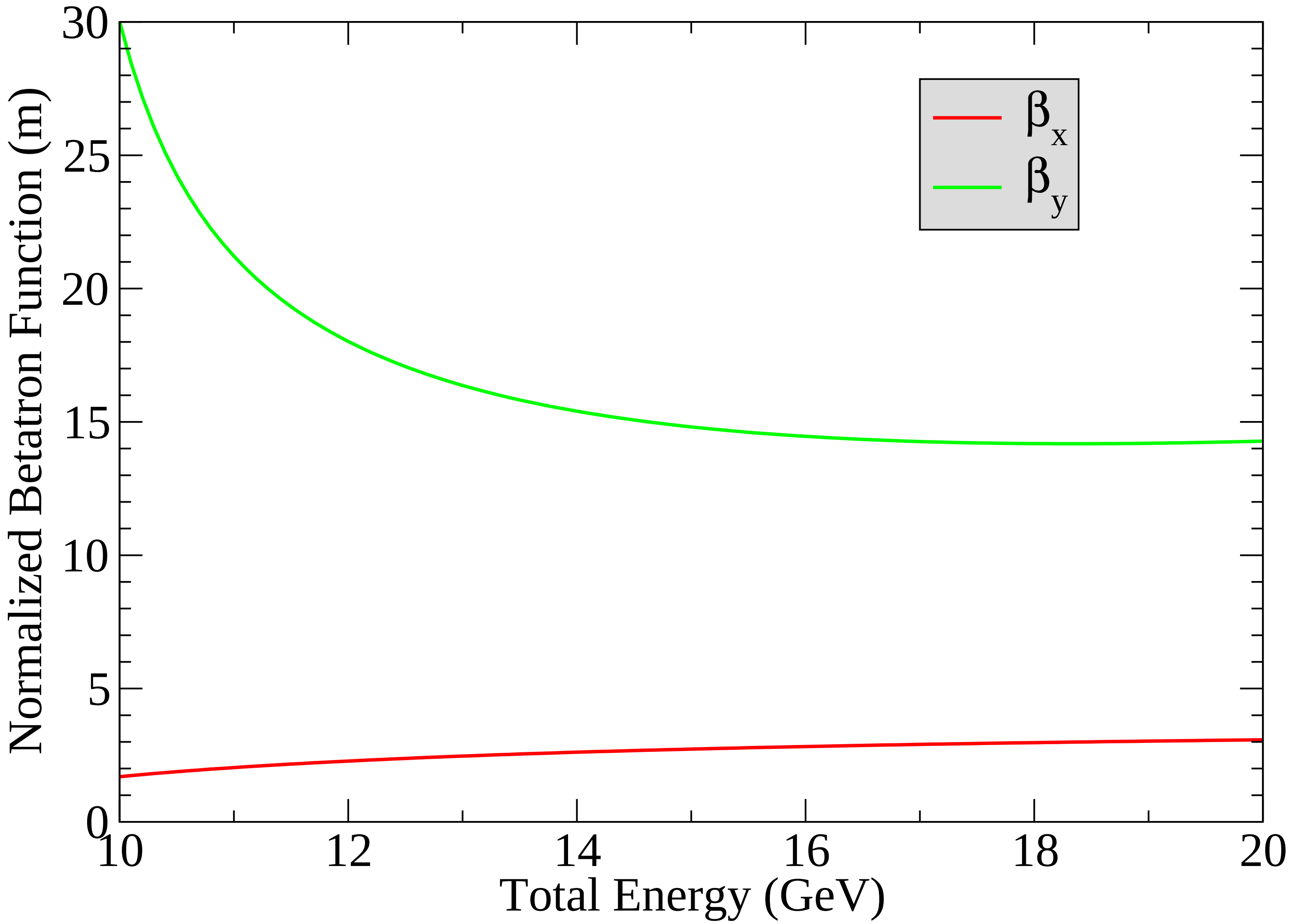
Min Energy (GeV)	5			10		
Max Energy (GeV)	10			20		
$V/\omega\Delta T\Delta E$	1/8			1/12		
Tunes	0.3	0.35	0.4	0.3	0.35	0.4
Cells	98	90	88	115	105	102
D Length (cm)	135	161	201	150	176	215
D Radius (cm)	12.3	14.1	17.3	9.2	10.4	12.8
D Pole Tip (T)	3.2	2.9	2.5	4.6	4.3	3.7
F Length (cm)	102	107	113	125	128	136
F Radius (cm)	15.4	15.3	16.2	12.9	12.6	13.3
F Pole Tip (T)	1.6	1.6	1.6	2.2	2.2	2.1
RF Voltage (MV)	735	675	660	863	788	765
$c\Delta T$ (cm)	27.9	25.6	25.0	24.5	22.4	21.8
Circumference (m)	627	607	640	805	768	803
Magnet cost (PB)	29	30	36	37	38	44
RF cost (PB)	44	40	39	52	47	46
Linear cost (PB)	16	15	16	20	19	20
Total cost (PB)	88	85	91	109	104	109

- Triplet
- Symmetrize parabola
- Add 8 empty cells for injection/extraction
- Try different low-energy tunes
- Higher tune, lower voltage
- Higher tune, D quad size rises
  - Spike in beta at low energy
  - More integrated quad strength
- Optimum tune in middle, 0.35
- Cost improved from before: symmetrized parabola

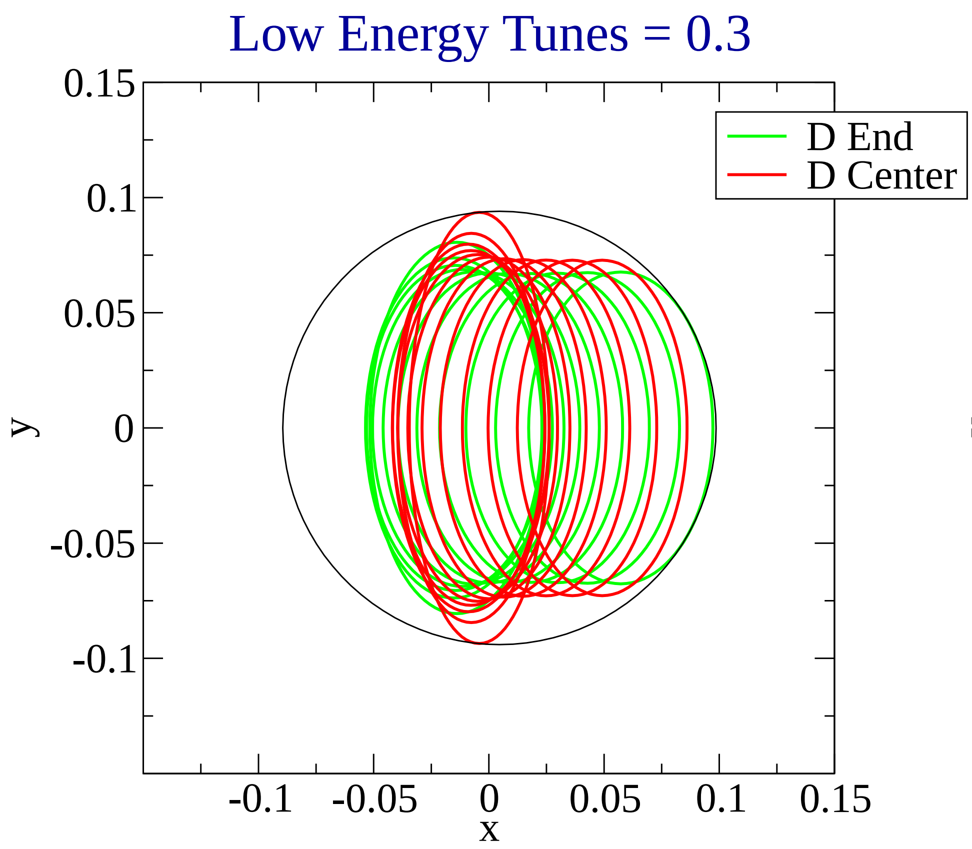
Symmetrized Time-of-Flight Parabola

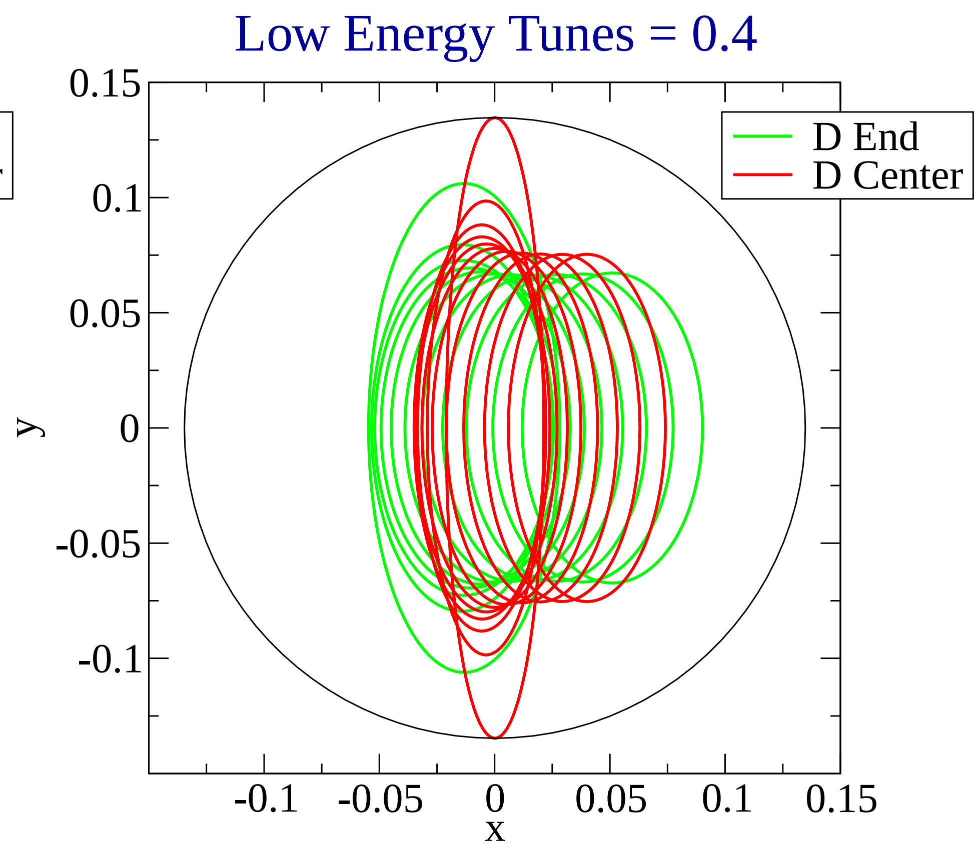


Beta Function vs. Energy



Fixed Tunes: Ellipses



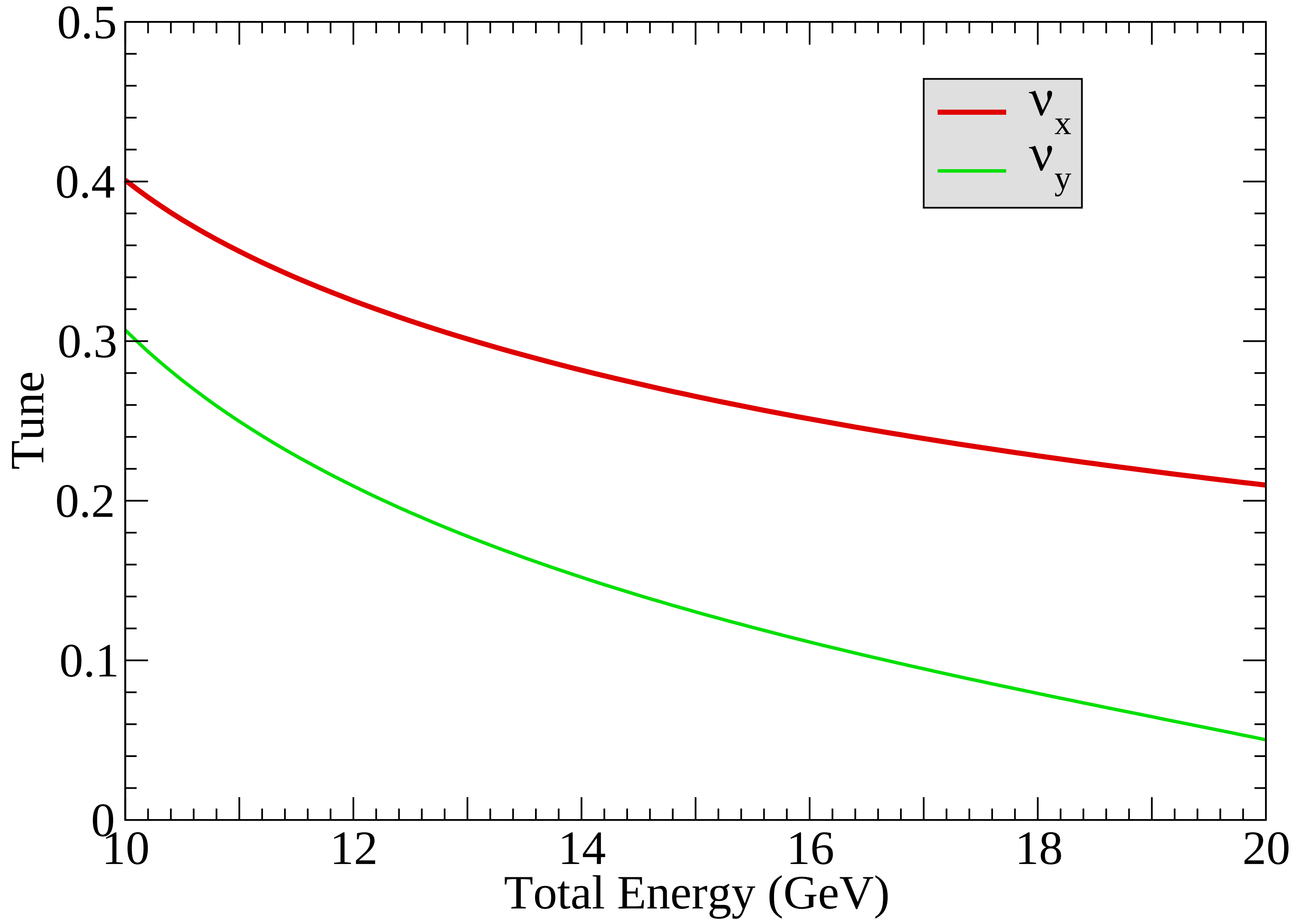


Tunes not Fixed, Cost Optimized

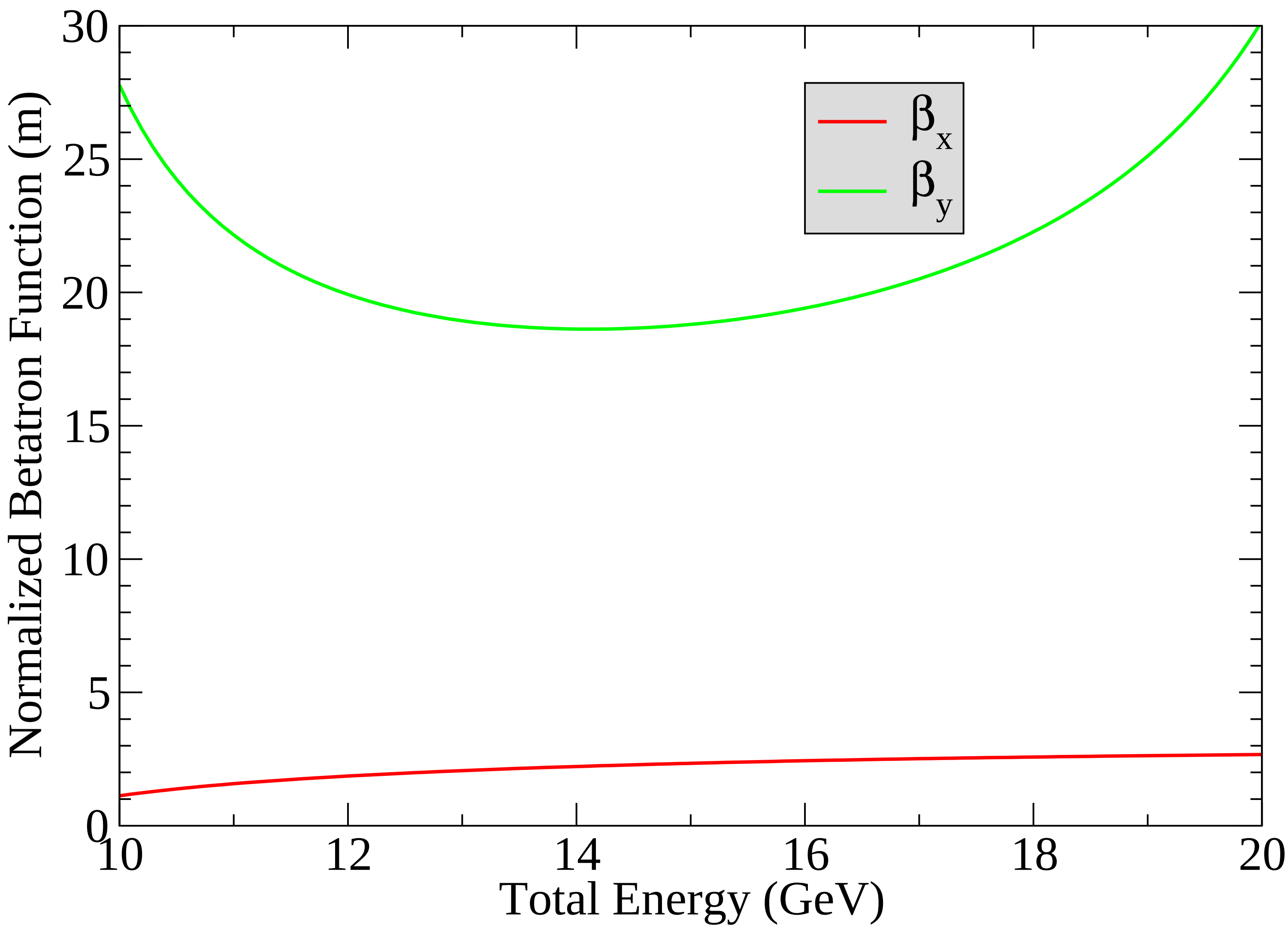
Min Energy (GeV)	5	10
Max Energy (GeV)	10	20
$V/\omega\Delta T\Delta E$	1/8	1/12
Cells	81	95
D Length (cm)	168	188
D Radius (cm)	13.6	10.3
D Pole Tip (T)	2.7	3.8
F Length (cm)	100	124
F Radius (cm)	14.9	12.4
F Pole Tip (T)	1.8	2.4
RF Voltage (MV)	608	713
$c\Delta T$ (cm)	23.0	20.3
Circumference (m)	542	699
Magnet cost (PB)	26	34
RF cost (PB)	35	42
Linear cost (PB)	14	17
Total cost (PB)	75	93

- Don't fix low-energy tunes
- Otherwise as before
- Cost, number of cells, circumference, voltage all down by about 10%
- Tunes at low energy split
  - Horizontal higher
  - Horizontal determines time-of-flight range, so higher better
  - Vertical lower to avoid spike in low-energy vertical beta function
    - Not too low: overall beta grows with lower tune
  - Optimally share integrated quad strength

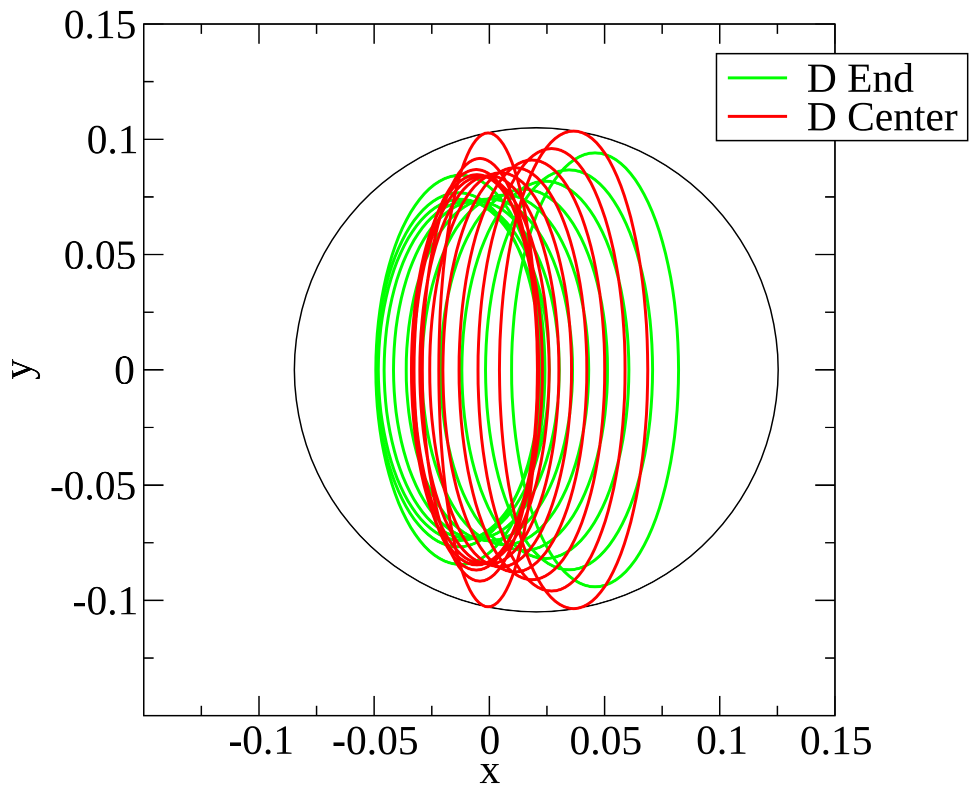
Split Tunes



Optimized Tunes: Beta Functions



Optimized Tunes: Ellipses



Conclusions

- Have a method for doing automated optimized lattice design for FFAGs
  - Gives precise answers, linearized about exact closed orbit
  - Great deal of flexibility (C++ code)
- Steady improvements in optimization have reduced triplet cost by 44%
  - Use lower pole tip fields (longer magnets)
  - Symmetrize parabola
  - Split tunes, with horizontal higher
- ΔT and/or circumference by themselves are not sufficient figures of merit
- Doublet may do better than triplet: need to analyze
- Similar methods apply to non-muon lattices, but conclusions may be different