Muon Acceleration in Doublet Lattice

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My WWW home directory:

`http://keil.home.cern.ch/keil/MuMu/Doc/FFAG_Apr04/doublet.pdf`

Longitudinal Dynamics

- Longitudinal Hamiltonian for stationary buckets

$$H_1(p_t, \varphi) \quad = \quad \frac{2\pi h \beta_0^2 E_0}{eVN_c} \left(\frac{\eta_0 p_t^2}{2} + \frac{\eta_1 p_t^3}{3} + \ldots \right) + \sin^2 \pi\varphi$$

  – p_t momentum error relative to reference particle with total energy E_0 and speed $\beta_0 c$
  – φ phase measured in cycles with origin at stable fixed point and $-1/2 \leq \varphi \leq +1/2$
  – h harmonic number, V peak accelerating voltage, N_c number of RF cavities

- Consider motion near transition with $\eta_0 = 0$, and $\eta_1 \neq 0$

Longitudinal Motion Near Transition

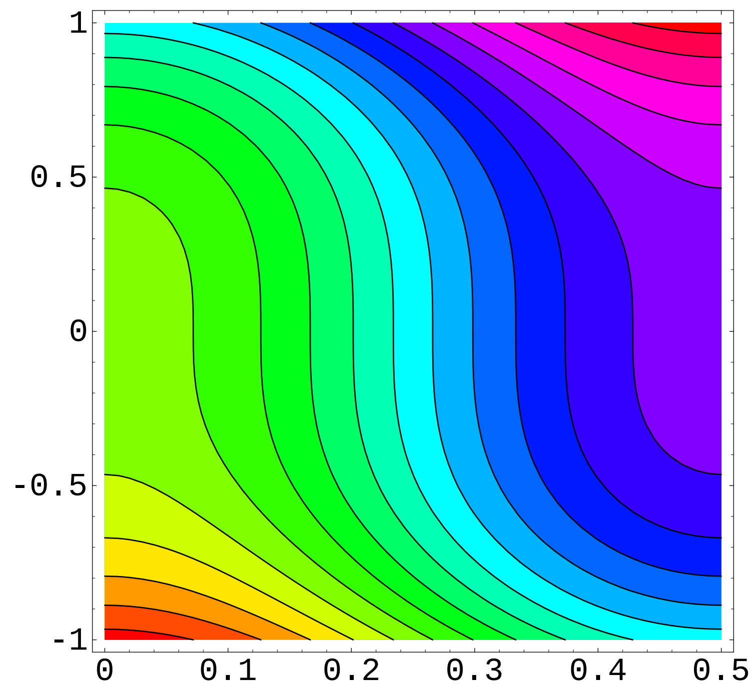
- Introduce scaled momentum variable y

$$y = p_t \left(\frac{2\pi\beta_0^2 E_0 h\eta_1}{3eVN_c} \right)^{1/3}$$

- Scaled Hamiltonian $H_5(y, \varphi)$

$$H_5(y, \varphi) = y^3 + \sin^2 \pi\varphi$$

- Acceleration in FFAG rings happens along light blue S-shaped trajectory, which starts at $\varphi = 1/2$ and $y = -1$, and reaches maximum $y = 1$ at $\varphi = 0$
- Equation relates range $\pm p_t$ and ring parameters at $y = \pm 1$, cf. next page



Contour plot of $H_5(y, \varphi)$

Parameters and Scaling Laws

- Calculate RF cavity voltage V from accelerating range p_t and ring parameters:

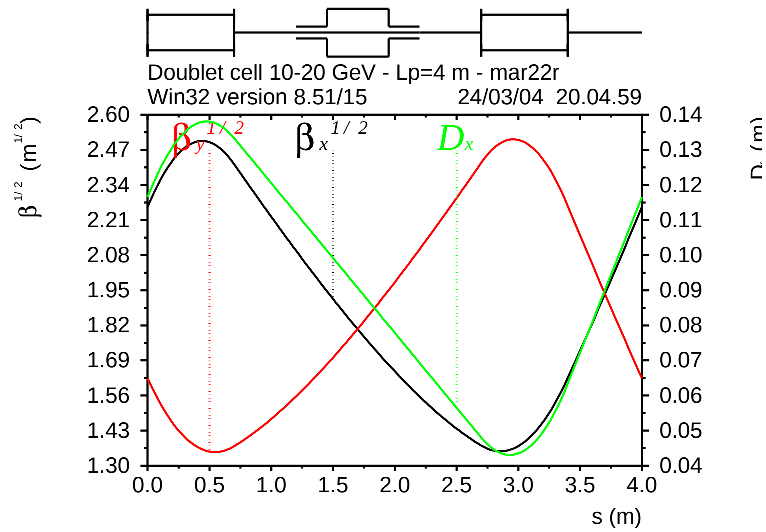
$$V = \frac{2\pi\beta_0^2 E_0}{3e}\left(\frac{h\eta_1}{N_c}\right)p_t^3$$

- Scaling with energy E_0 in first term, with range p_t in third term
- Scaling with N lattice periods of length L in brackets:
  - h and circumference C at given RF frequency $\propto LN$
  - $N_c \propto N$
  - $\eta_1 \propto 1/N^2$ derived analytically by K.Y. Ng for FODO lattice with $N \gg 1$; I believe from numerical studies that it holds for any lattice style
- $V \propto E_0 L p_t^3/N^2$ and $N_c V \propto E_0 L p_t^3/N$
- Assuming that cost of magnets, vacuum, tunnel is $C_\mathrm{M} LN$, that cost of RF cavities and power installation is $C_\mathrm{RF} E_0 L p_t^3/N$ yields cost optimum at eqal cost components

$$C = 2L\sqrt{C_\mathrm{M} C_\mathrm{RF} E_0 p_t^3}$$

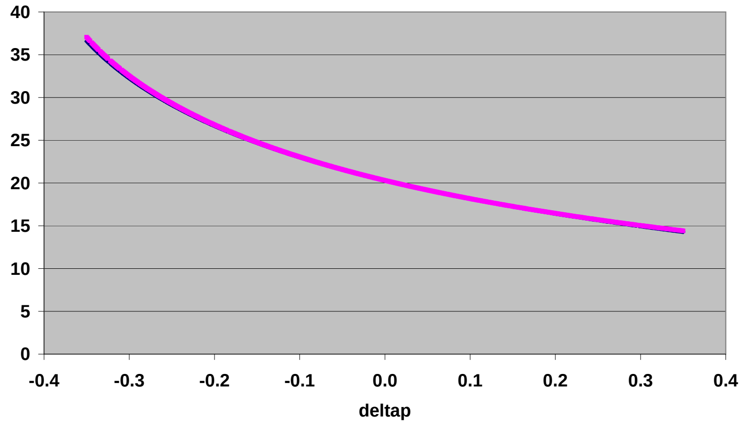
Keil-Sessler Doublet Lattice

- Horizontally focusing gradient dipoles
- Vertically focusing gradient dipoles
- FODO lattice with $Q_x \approx Q_y$
- Space for super-conducting RF cavity

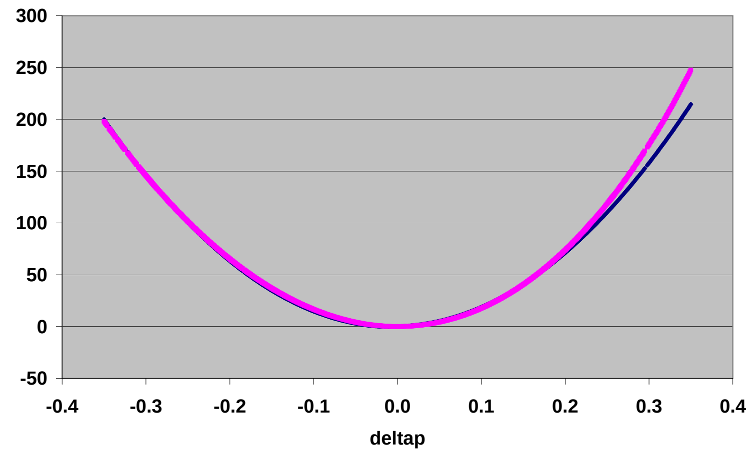


Graphs for Keil-Sessler Doublet Lattice

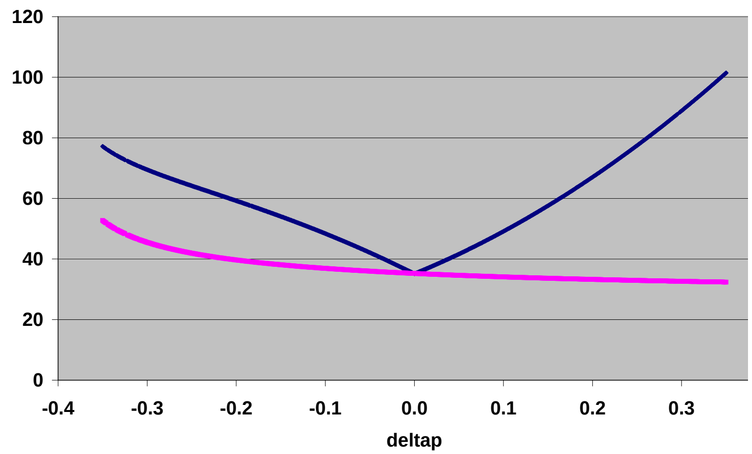
Tunes q_x and q_y vs. $\delta p/p$



Orbit length $\delta(s)$ and travel time ct in mm vs. $\delta p/p$



Horizontal aperture A_x and vertical aperture A_y in mm vs. $\delta p/p$

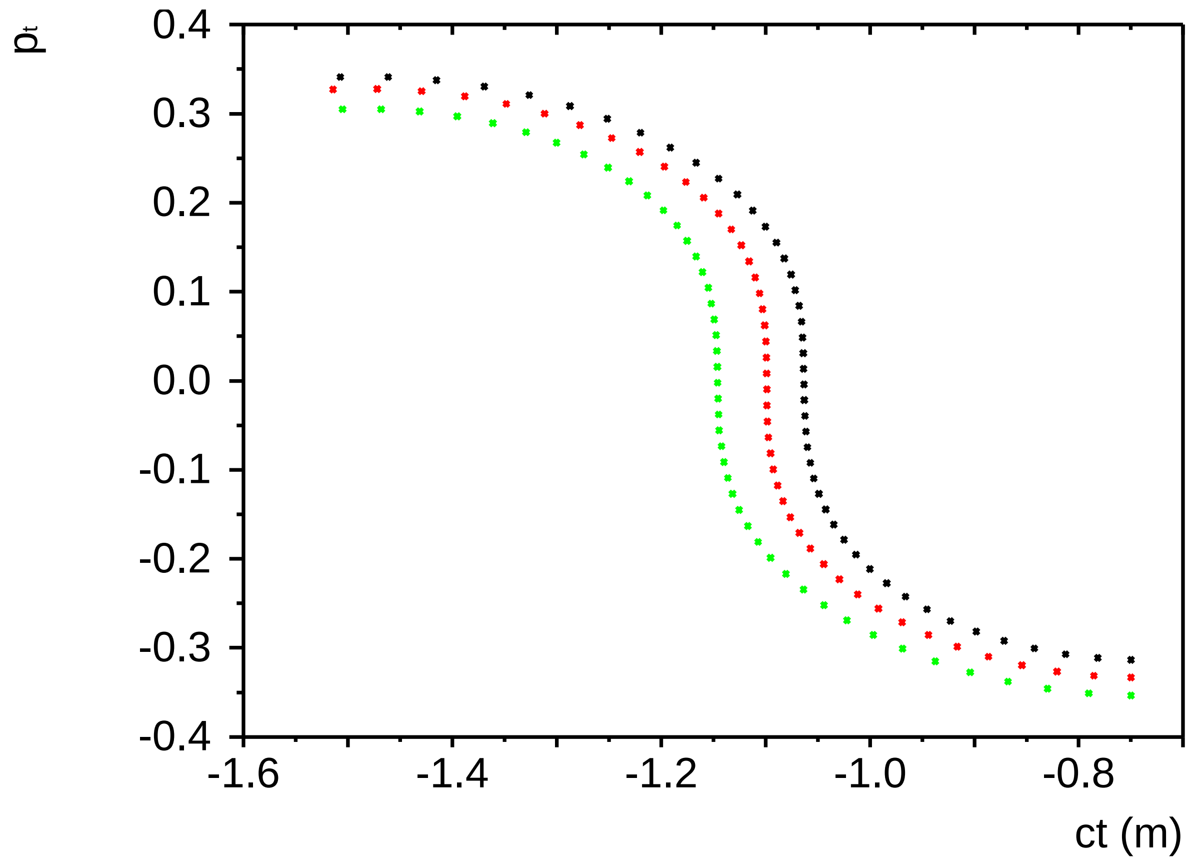


- Stable tunes q_x and q_y in range $-1/3 \leq \delta p/p \leq 1/3$
- Aperture radii $A_x < 100$ mm and $A_y < 50$ mm for $\varepsilon_n = 5/3$ mm
- Fit to ct yields $\eta_1 = 0.004233$

Lattice Parameters of Keil-Sessler Doublet Lattice

Total ref. energy E/GeV	15
Energy range/GeV	$10\ldots20$
Offset in F magnet x/mm	$-25\ldots69$
Period length L_p/m	4
Periods N_p	100
Circumference C/m	400
Gradients G_F/G_D/T/m	52.6/−52.6
Dipole field B_F/B_D/T	−2.4/6.9
Path length spread/mm	221
Slip factor η_0	0
Slip factor η_1	0.004233

Acceleration in Keil-Sessler Doublet Lattice



RF System Parameters of Keil-Sessler Doublet Lattice

Range of p_t	$\pm 1/3$
RF frequency/MHz	202.4
Number of RF cavities	100
RF cavity voltage V/MV	13.5
Circumf. RF accel. voltage	1350
Number of turns	9