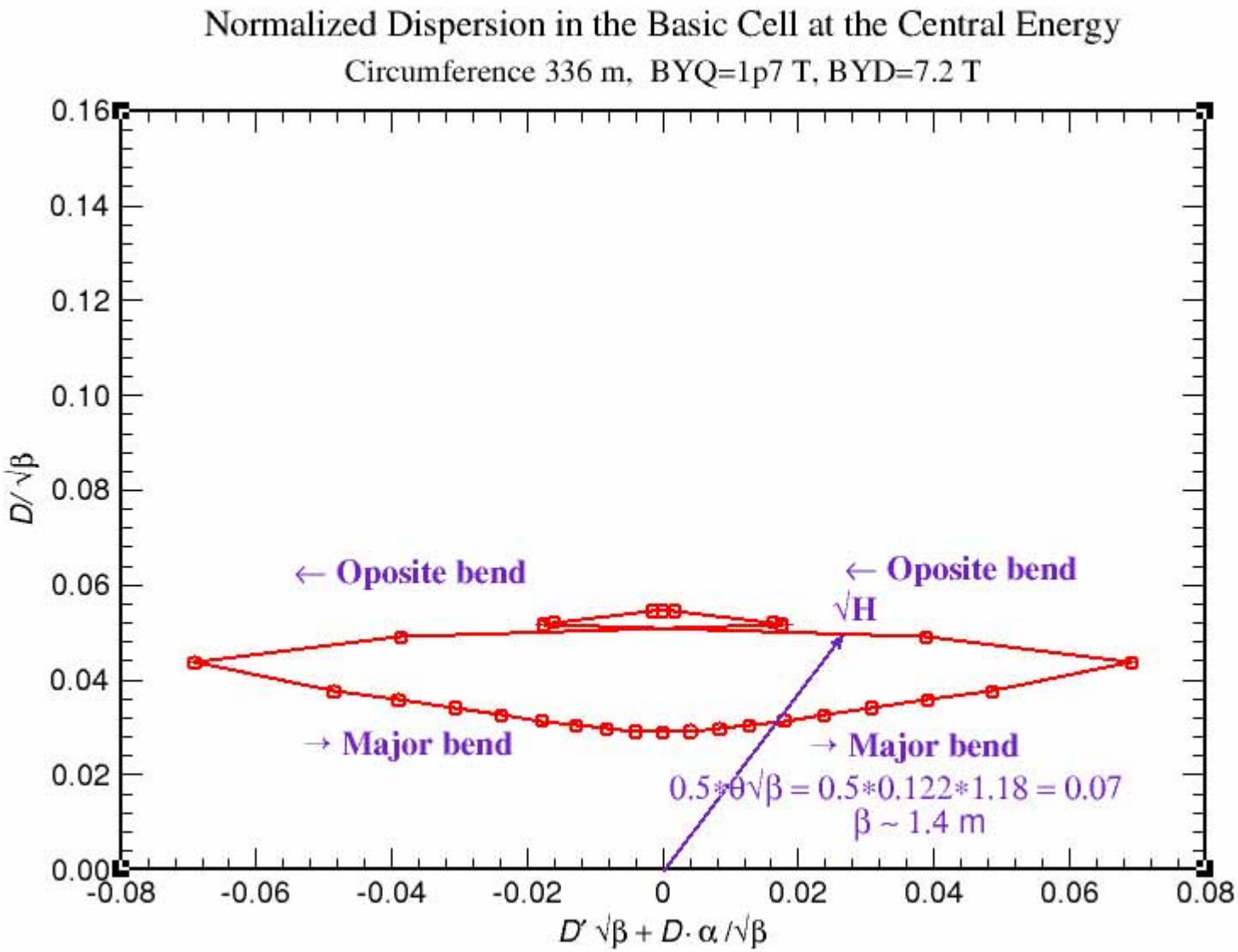
Non-Scaling FFAG for:
the proton acceleration: 200 MeV – 1.5 GeV,
KEK muon acceleration 10-20 GeV, and
electron acceleration in eRHIC

- **Basic cell:**
  - **Betatron functions at the central energy**
  - **required bending fields and gradients**
- **Orbits at each energy PTC results**
- **Tunes vs. momentum**
- **Maximum orbit offsets**
- **Difference in the path lengths at each momentum during acceleration.**


Normalized Dispersion in the Basic Cell at the Central Energy
Circumference 336 m, BYQ=1p7 T, BYD=7.2 T
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
D/√β
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
D′ √β + D· α /√β
← Oposite bend
← Oposite bend
√H
→ Major bend
→ Major bend
0.5*θ√β = 0.5*0.122*1.18 = 0.07
β ~ 1.4 m

Scaling or non- scaling FFAG, Minimum emittance lattice or FODO?

Scaling FFAG properties:

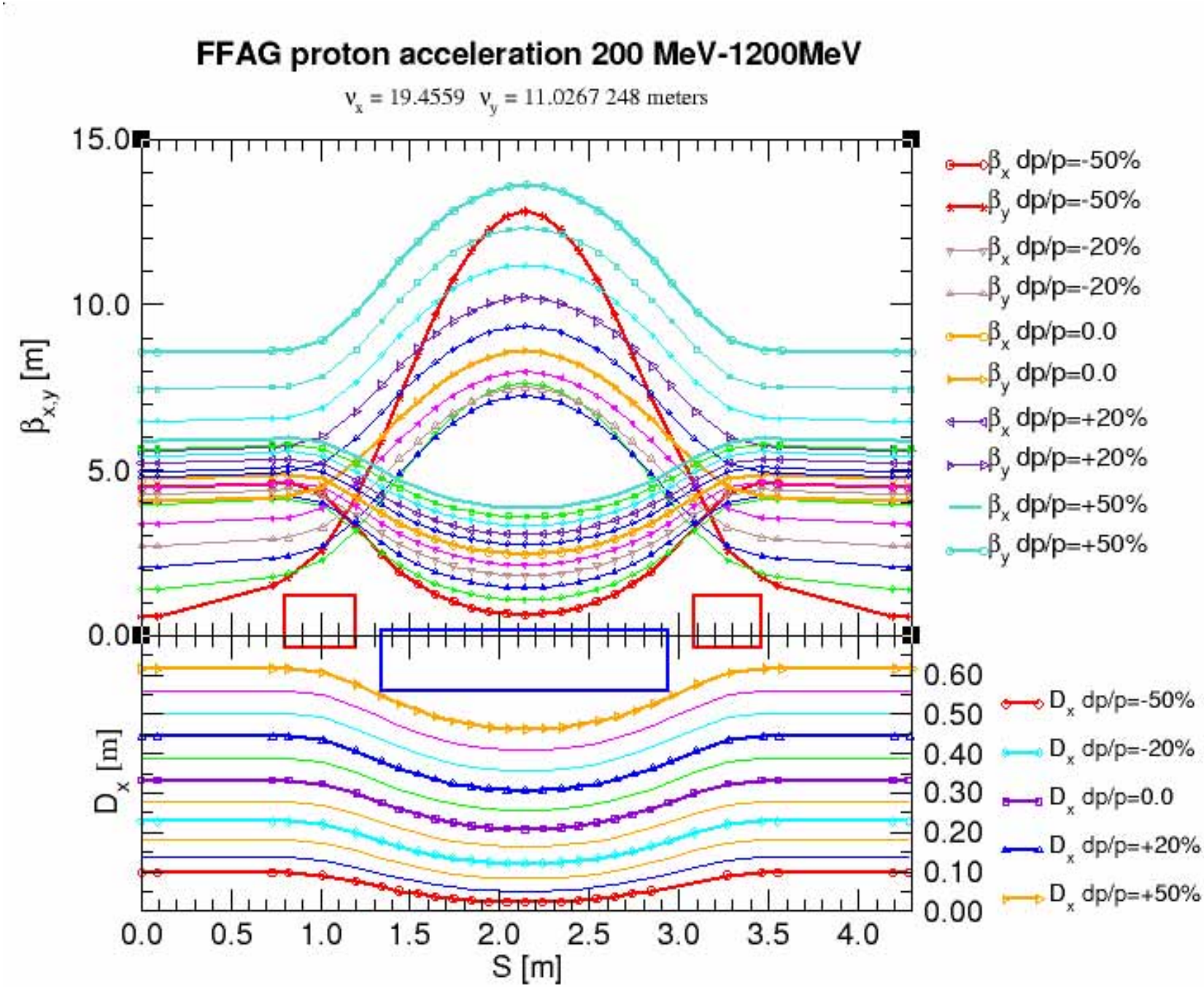
- Zero chromaticity.
- Orbits parallel for different energies.
- Large momentum acceptance.
- Relatively large circumference.
- Relatively large physical aperture.
- RF:large aperture-follows the energy.
- Tunes are fixed for all energies.
- Negative momentum compaction.
- Orbits of the high energy particles are at high field, low energy particles at low field.

Minimum emittance FFAG properties:

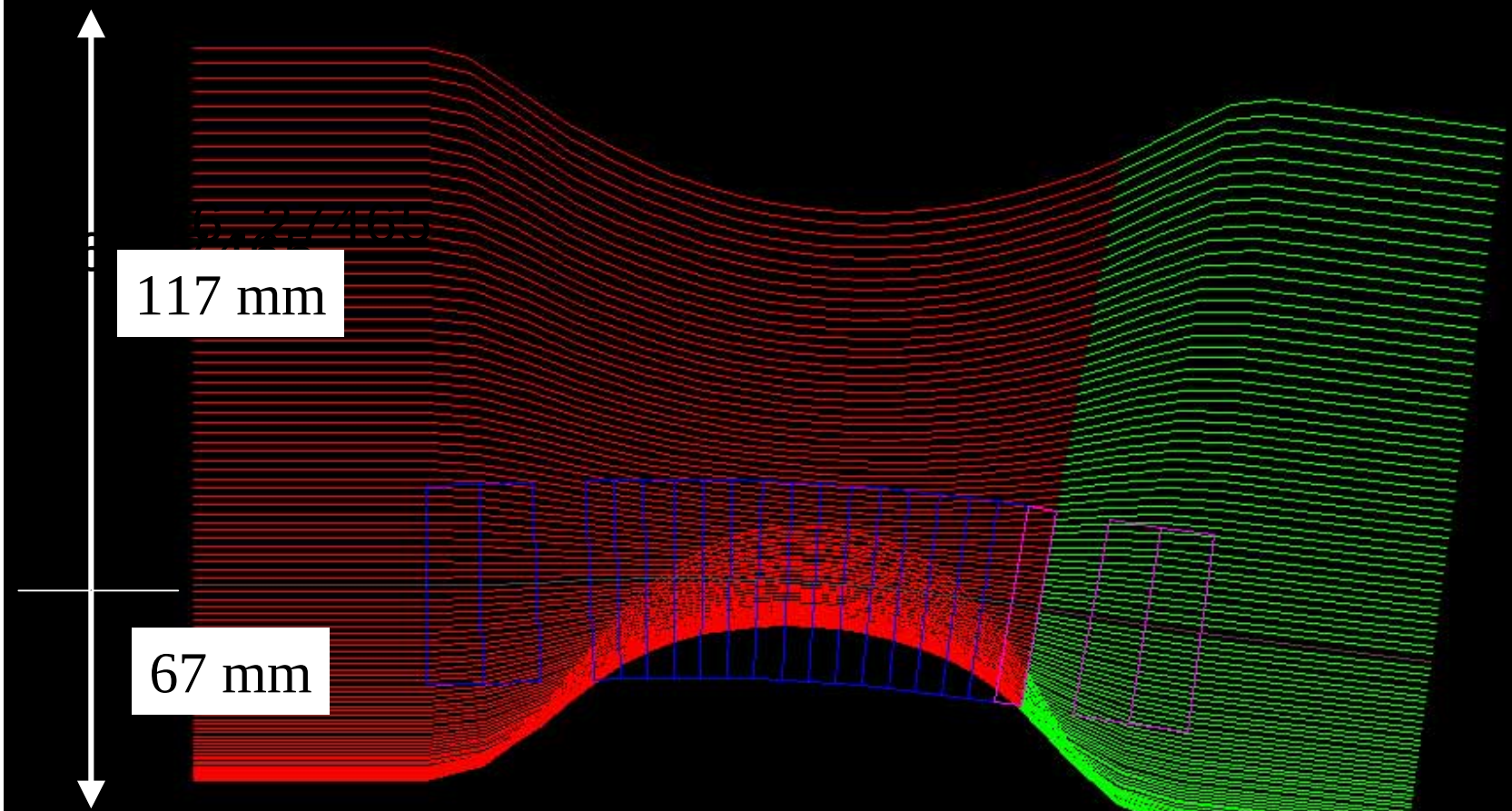
- Chromaticity is changing.
- Orbits not parallel.
- Large momentum acceptance.
- Relatively small circumference.
- Relatively small physical aperture.
- RF:small aperture-at the crest.
- Tunes move 0.4->0.1 in basic cell.
- Momentum compaction changes.
- Orbits of the high energy particles are at high field, low energy particles at low field.

FODO or minimum emittance lattice?

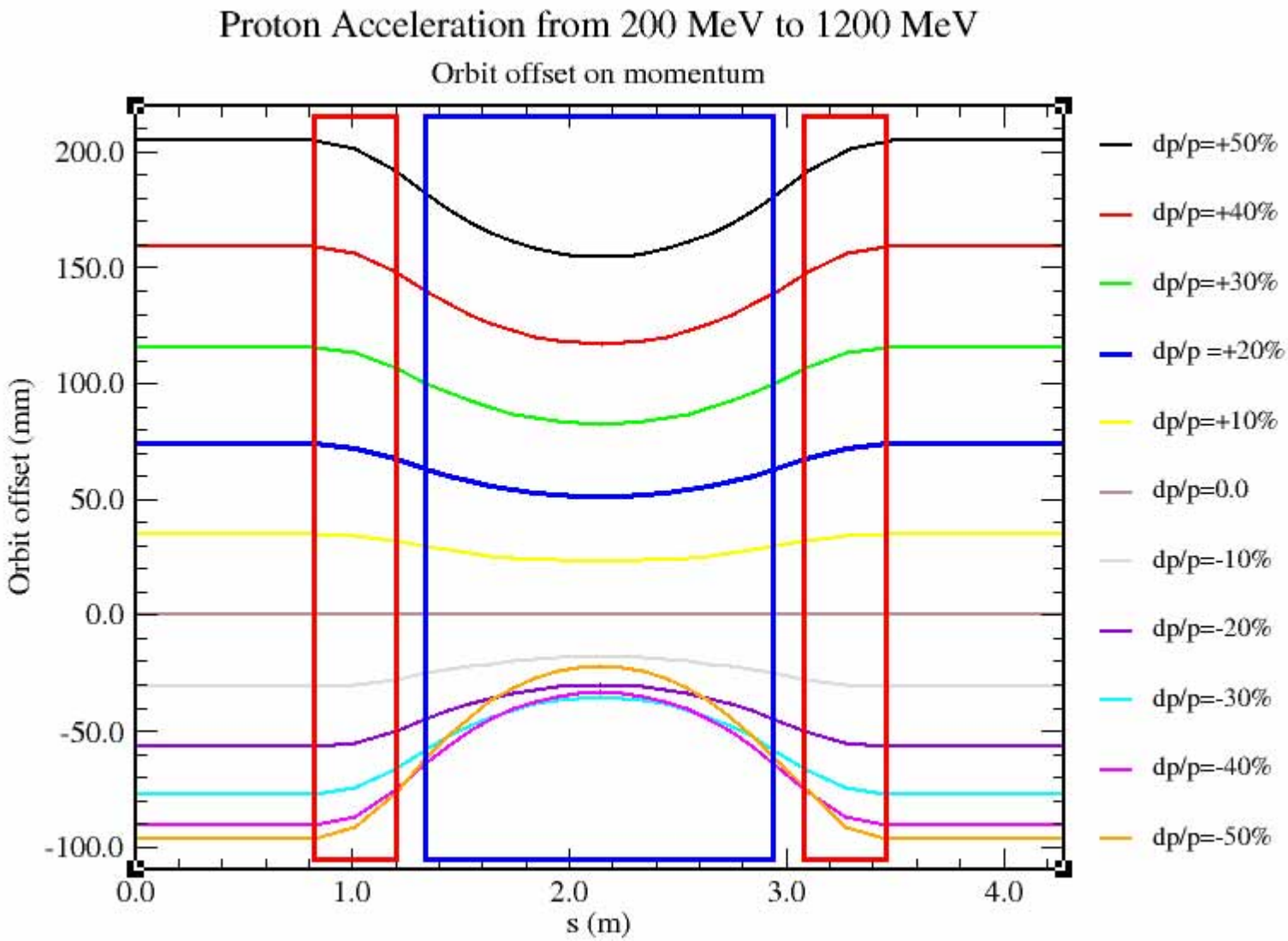
- For the same magnet properties larger circumference and larger X_{co}.
- For the same dispersion [$\Delta x = D_X * dp/p$] and the same magnet smaller field and larger circumference.
- The FODO has larger available free space.


FFAG proton acceleration 200 MeV-1200MeV
ν_x = 19.4559 ν_y = 11.0267 248 meters
$\beta_{x,y}$ [m]
D_x [m]
S [m]
β_x dp/p=-50%
β_y dp/p=-50%
β_x dp/p=-20%
β_y dp/p=-20%
β_x dp/p=0.0
β_y dp/p=0.0
β_x dp/p=+20%
β_y dp/p=+20%
β_x dp/p=+50%
β_y dp/p=+50%
D_x dp/p=-50%
D_x dp/p=-20%
D_x dp/p=0.0
D_x dp/p=+20%
D_x dp/p=+50%

Proton Acceleration 0.2-1.5 GeV

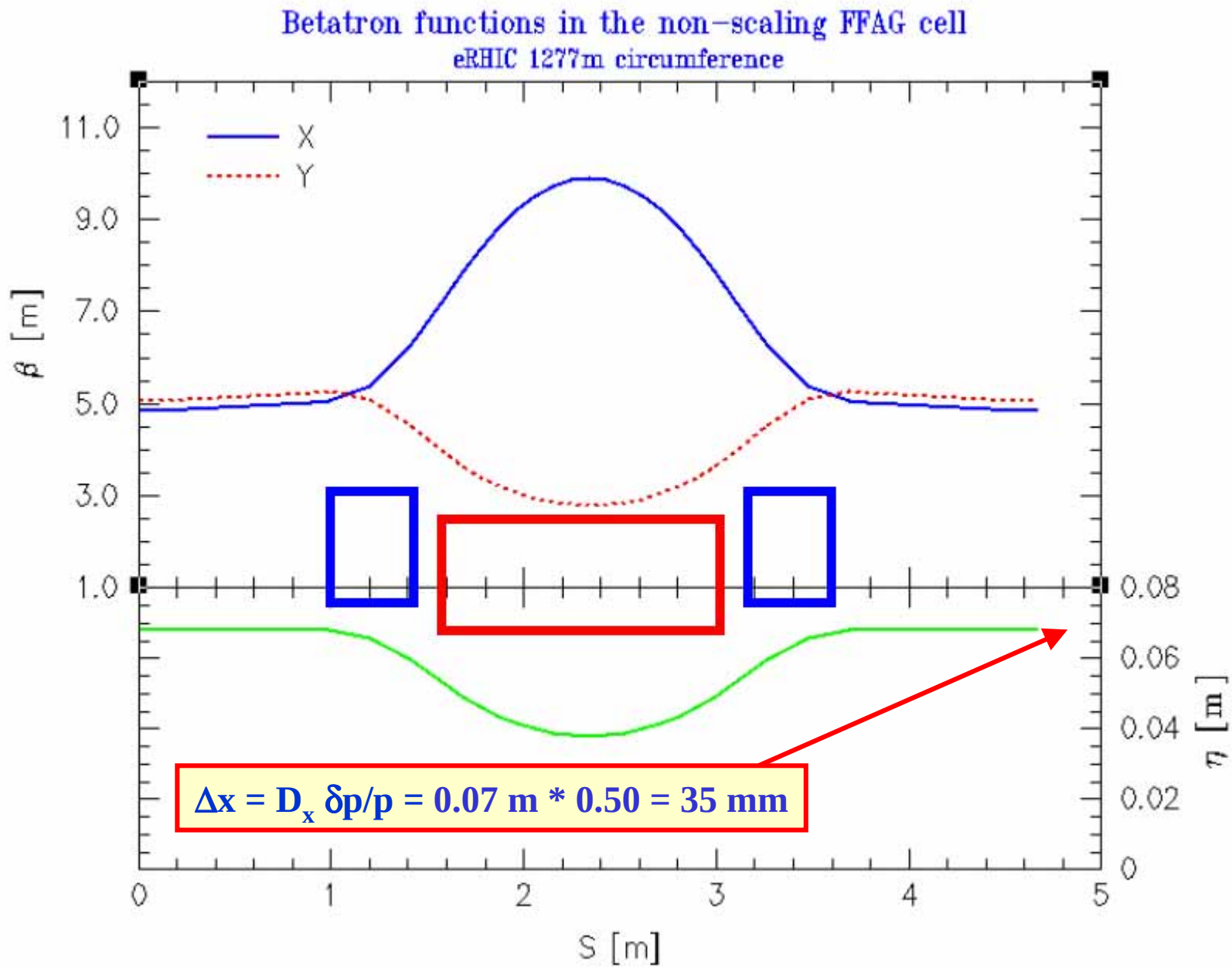


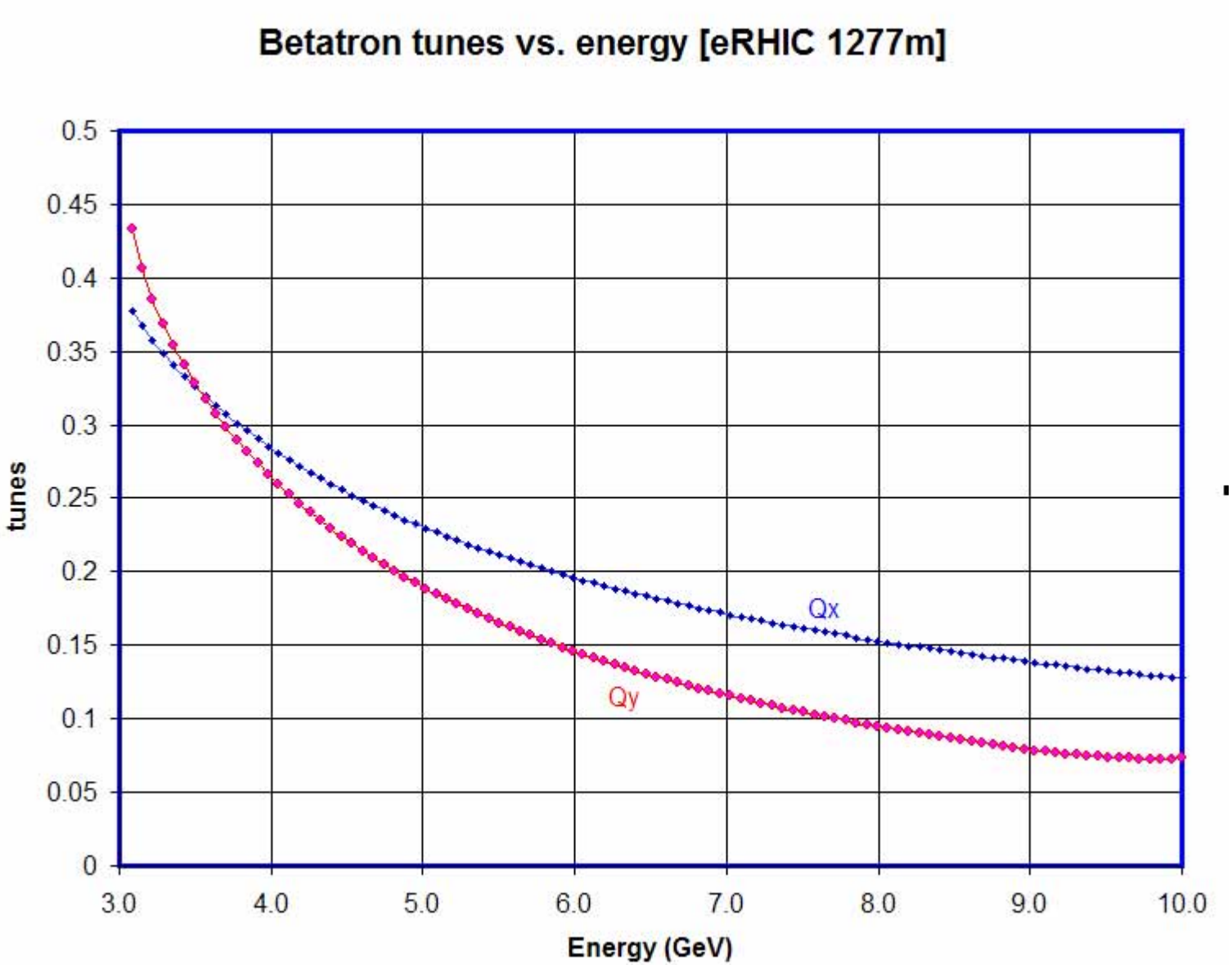
dp/p = -52 % - +52 %

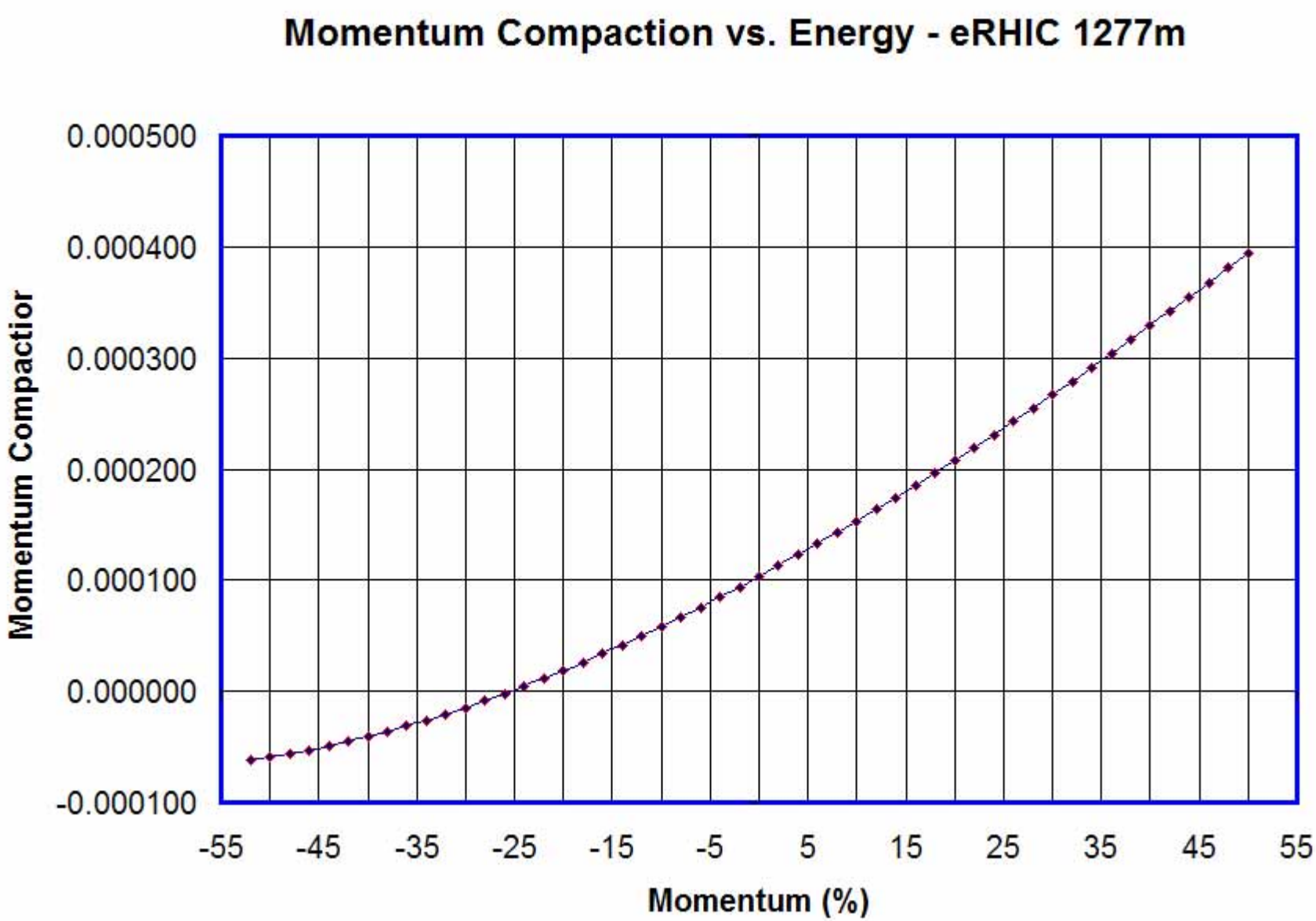

Proton Acceleration from 200 MeV to 1200 MeV
Orbit offset on momentum
Orbit offset (mm)
200.0
150.0
100.0
50.0
0.0
-50.0
-100.0
0.0
1.0
2.0
3.0
4.0
s (m)
dp/p=+50%
dp/p=+40%
dp/p=+30%
dp/p =+20%
dp/p=+10%
dp/p=0.0
dp/p=-10%
dp/p=-20%
dp/p=-30%
dp/p=-40%
dp/p=-50%

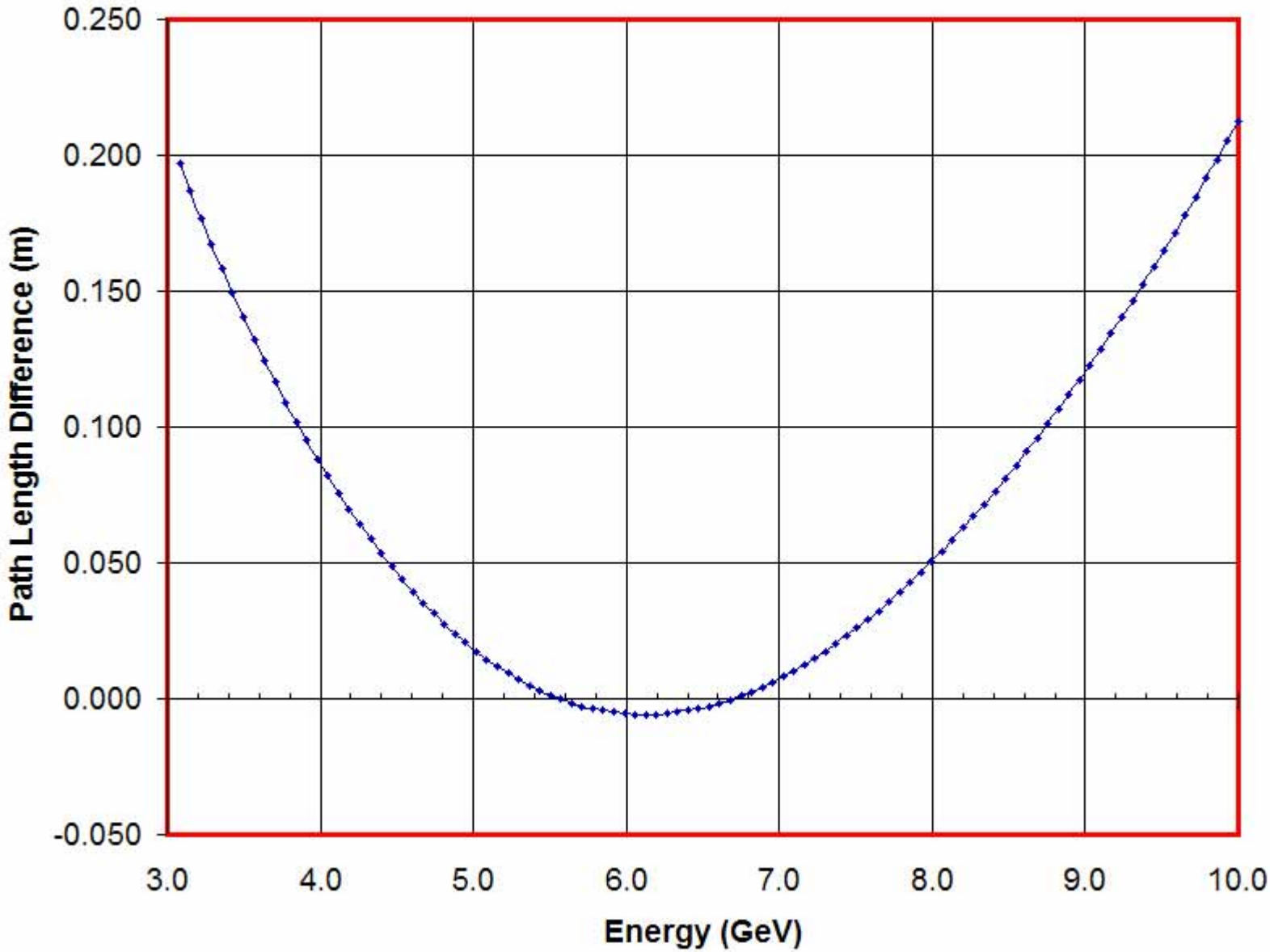
Electron Acceleration from 3-10 GeV with a non-scaling FFAG lattice

- **eRHIC non-scaling triplet FFAG lattice**
  - Courant-Snyder functions in the basic cell at the central energy, magnet lengths and required apertures.
  - Courant-Snyder functions during acceleration:
    - Orbit offsets
    - Tunes vs. energy
    - Amplitude functions vs. energy
    - Momentum compaction vs. energy
    - Path Length variation vs. energy
  - Synchrotron Radiation and requirements for the RF cavities.


Betatron functions in the non-scaling FFAG cell
eRHIC 1277m circumference
X
Y
β [m]
11.0
9.0
7.0
5.0
3.0
1.0
0.08
0.06
0.04
0.02
0
η [m]
0
1
2
3
4
5
S [m]
Δx = D_x δp/p = 0.07 m * 0.50 = 35 mm


Betatron tunes vs. energy [eRHIC 1277m]
tunes
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
Energy (GeV)
Qx
Qy


Momentum Compaction vs. Energy - eRHIC 1277m
Momentum Compactior
0.000500
0.000400
0.000300
0.000200
0.000100
0.000000
-0.000100
-55
-45
-35
-25
-15
-5
5
15
25
35
45
55
Momentum (%)

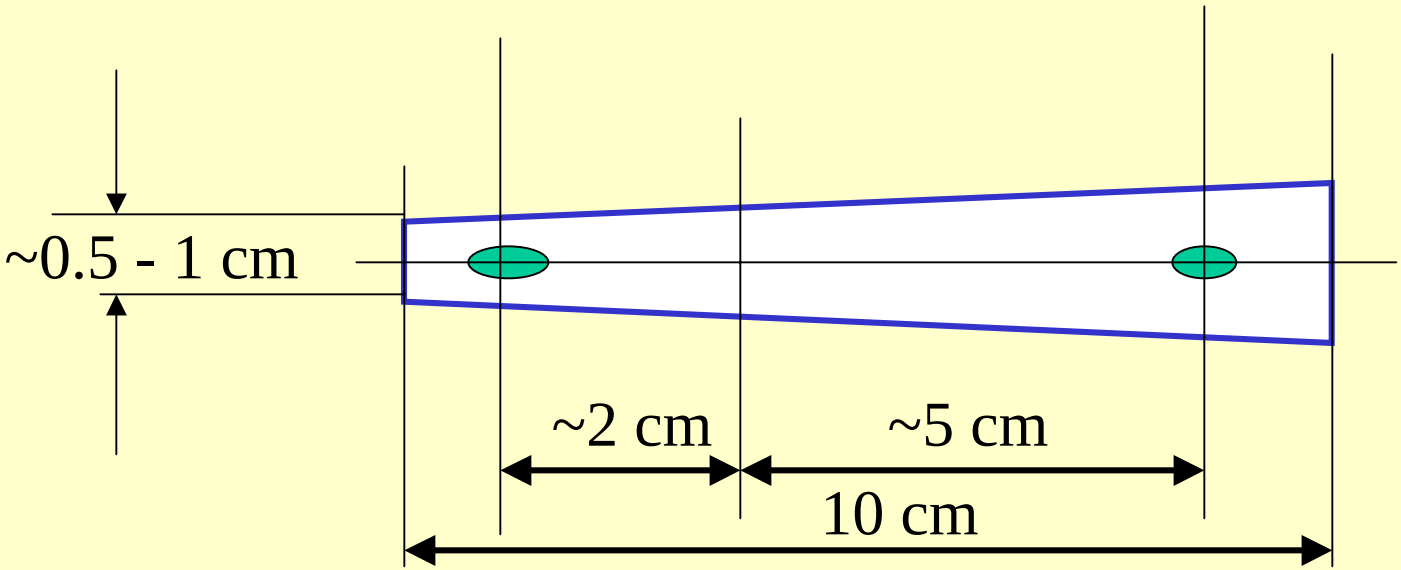

eRHIC FFAG acceleration 1277m
Path Length Difference (m)
0.250
0.200
0.150
0.100
0.050
0.000
-0.050
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
Energy (GeV)

Magnet apertures and fixed fields:

$\varepsilon = 5\ \pi$ mmmrad

$\sigma^2 = (11\text{m} * 5\ \pi\ \text{mmmrad} / 6\ \pi\ \beta\gamma)$

$\sigma_{3\text{GeV}} \cong 16\ \mu\text{m}$

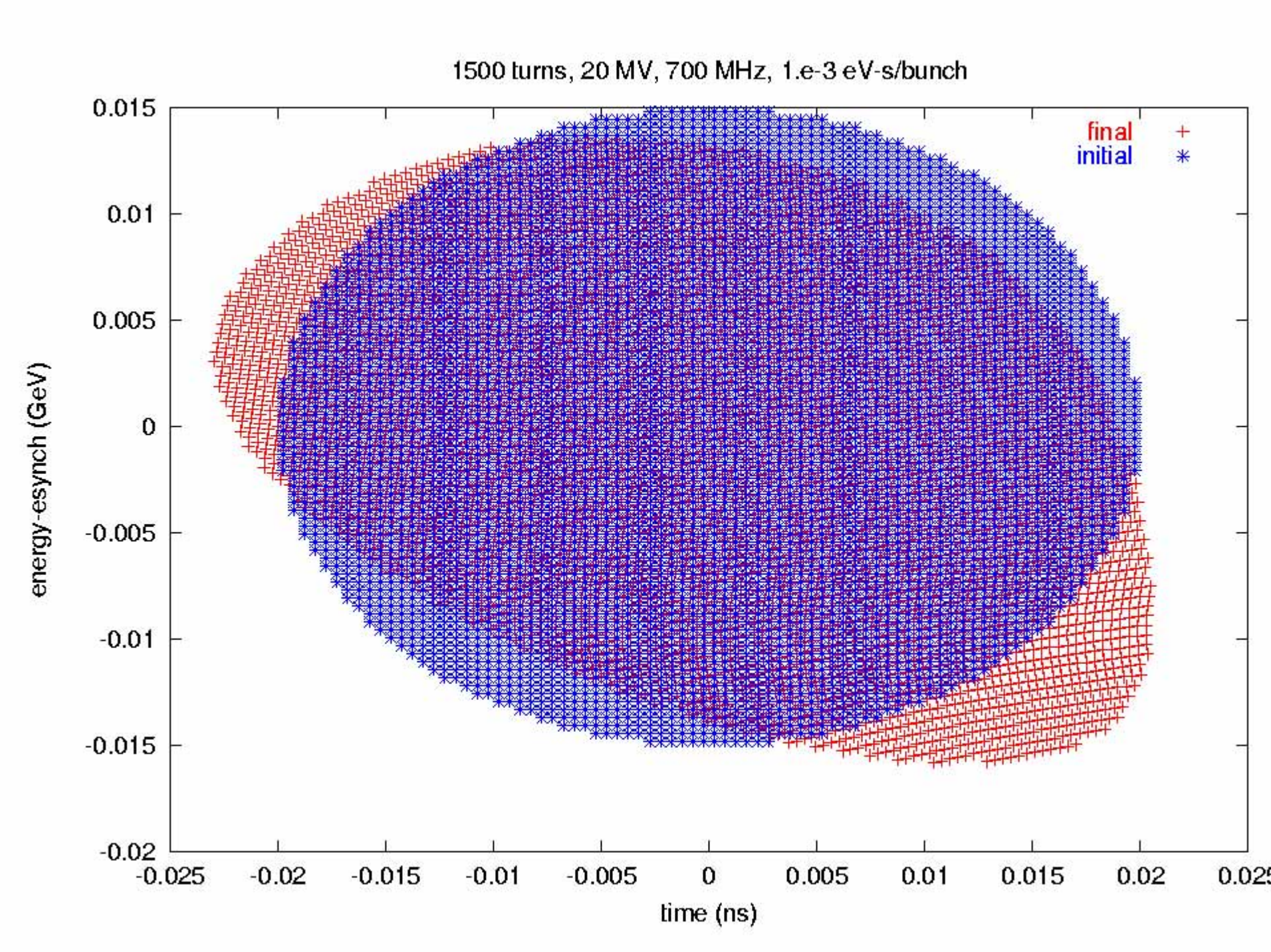


$B_{MB} = 0.52$ T

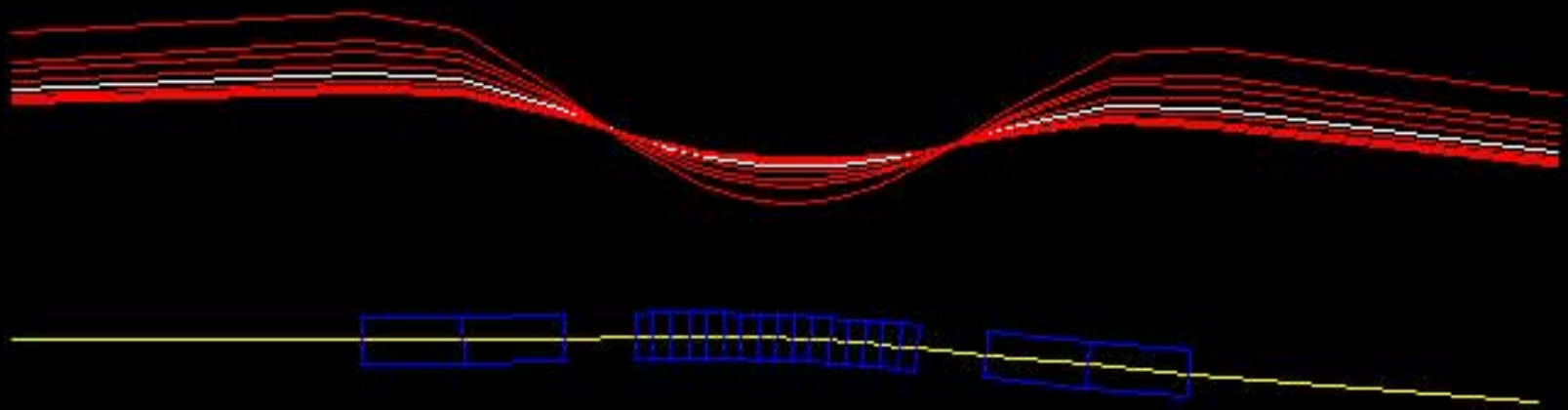
$B_{OPP} = 0.36$ T

$G_F = 0.36\ \text{T} / 1.68\ \text{cm} = 21.2\ \text{T/m}$

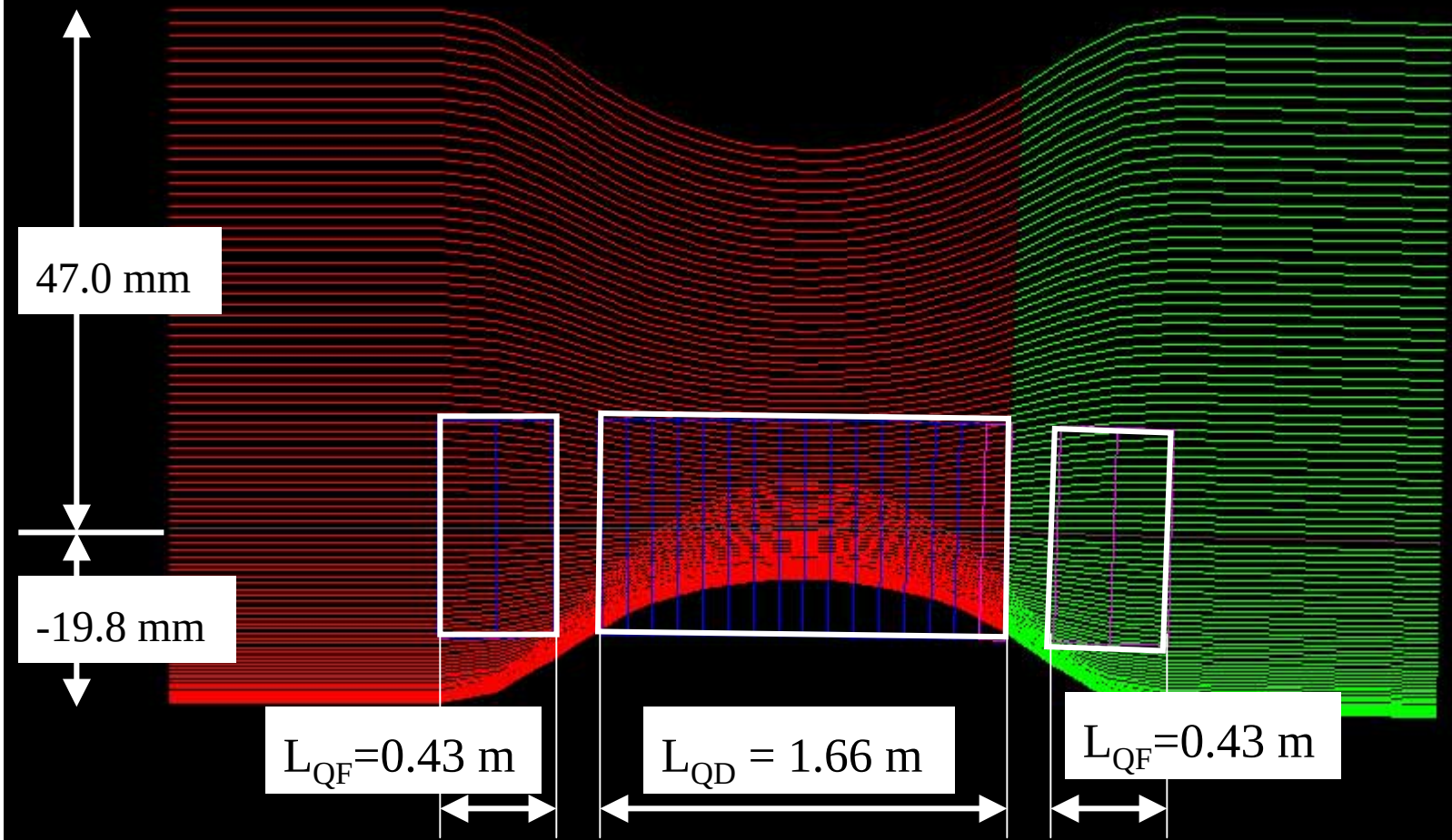
$G_D = 10.1\ \text{T/m}$


1500 turns, 20 MV, 700 MHz, 1.e-3 eV-s/bunch
final
initial
energy-esynch (GeV)
time (ns)
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02
0.025

Betatron Function during Acceleration



Electron path during acceleration within the basic cell
C=1277 m, 273 cells, L=4.68 m

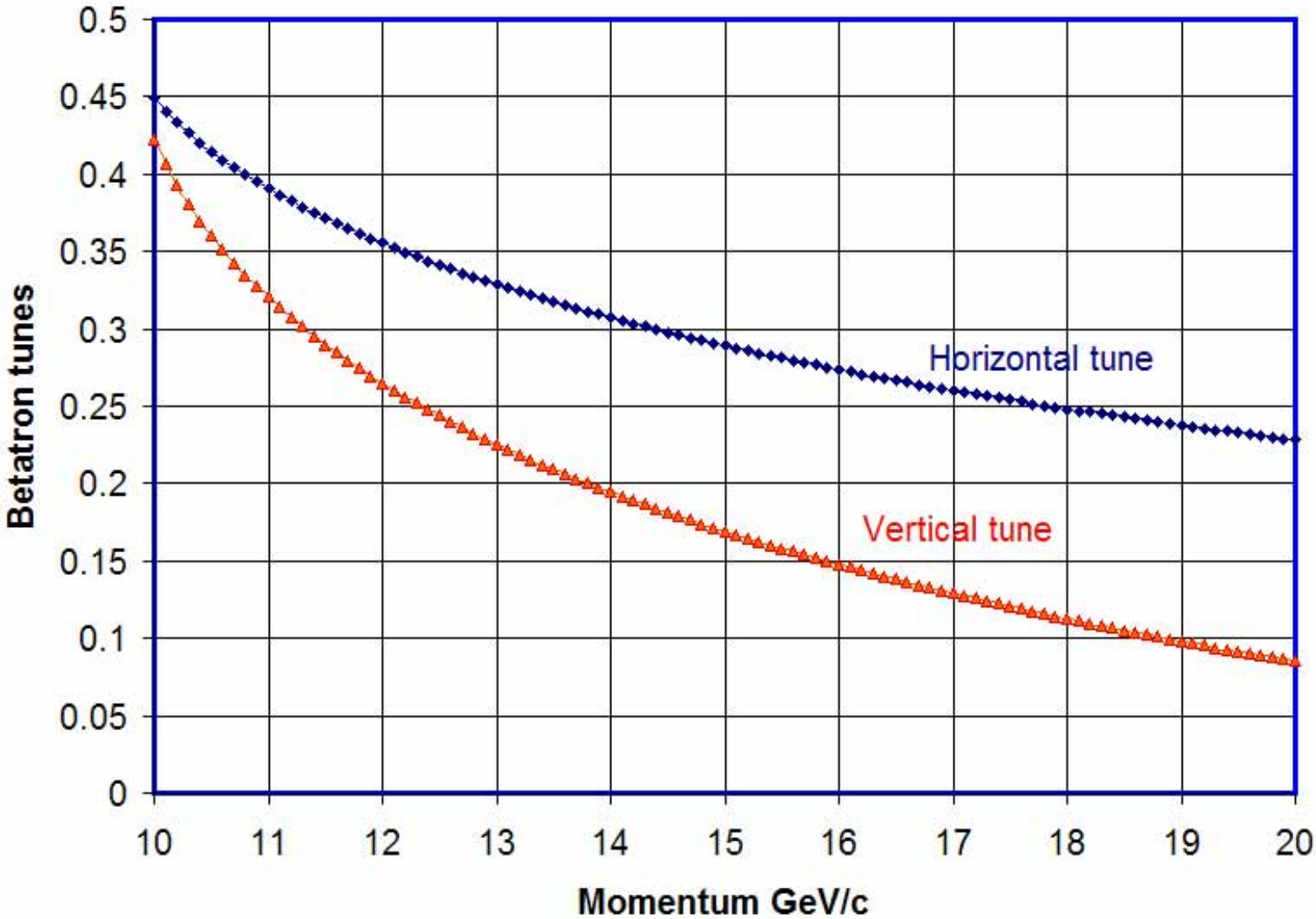


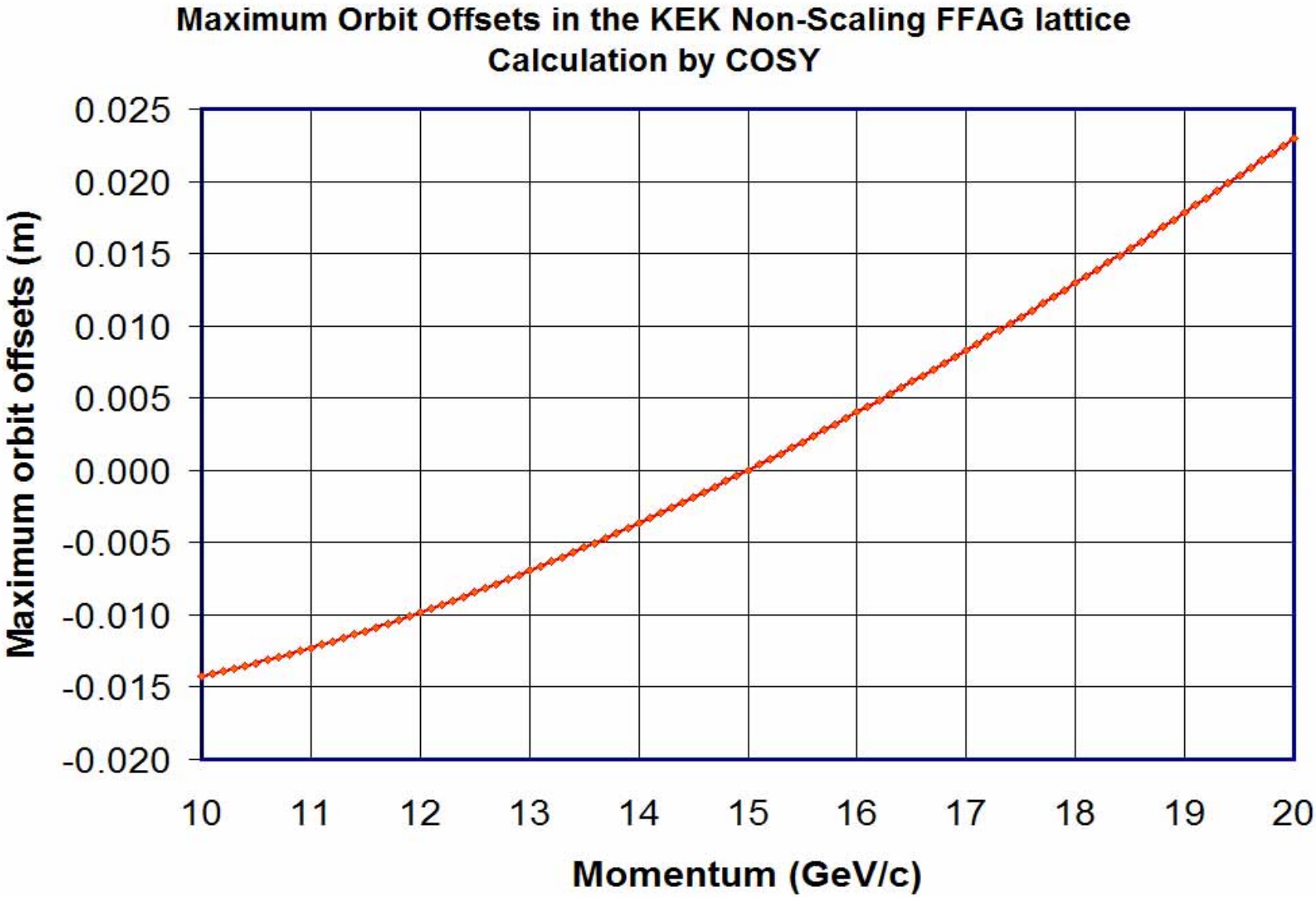
KEK lattice design for 10-20 GeV muon acceleration

```
QLF     =    0.75 m    opposite bend FOCUSING QUAD GF1
BL      =    1.80 m    Major Bend DEFOCUSING QUAD  GDD1
NDIP    =    195
```

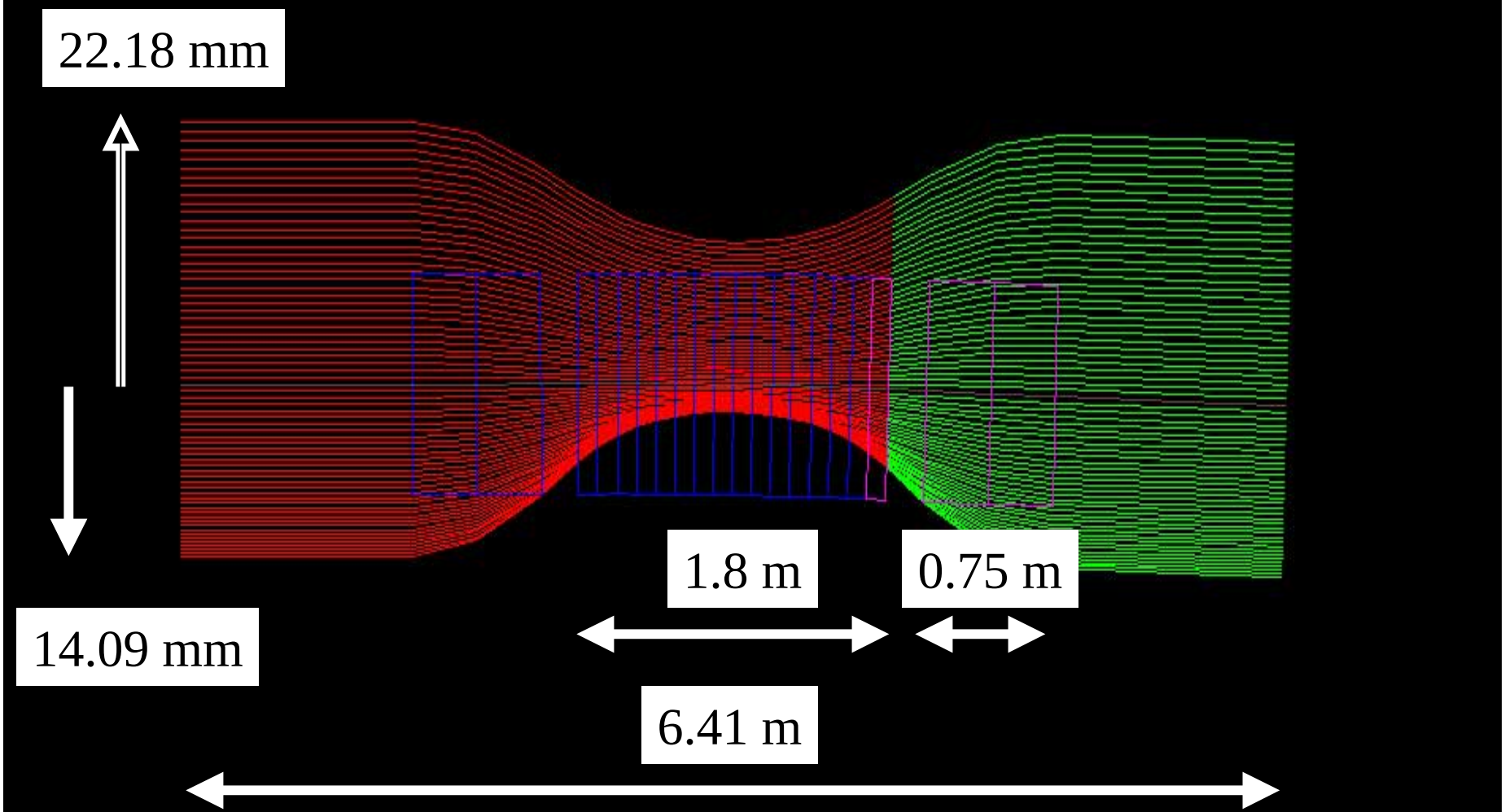
```
        FINAL VALUES OF VARIABLES
           1    GF1      1        0.34706360E+02
           2    GDD1     1       -0.23008450E+02
           3    D2P      1        0.34710655E+00
           4    D3P      1        0.21802168E+00
```

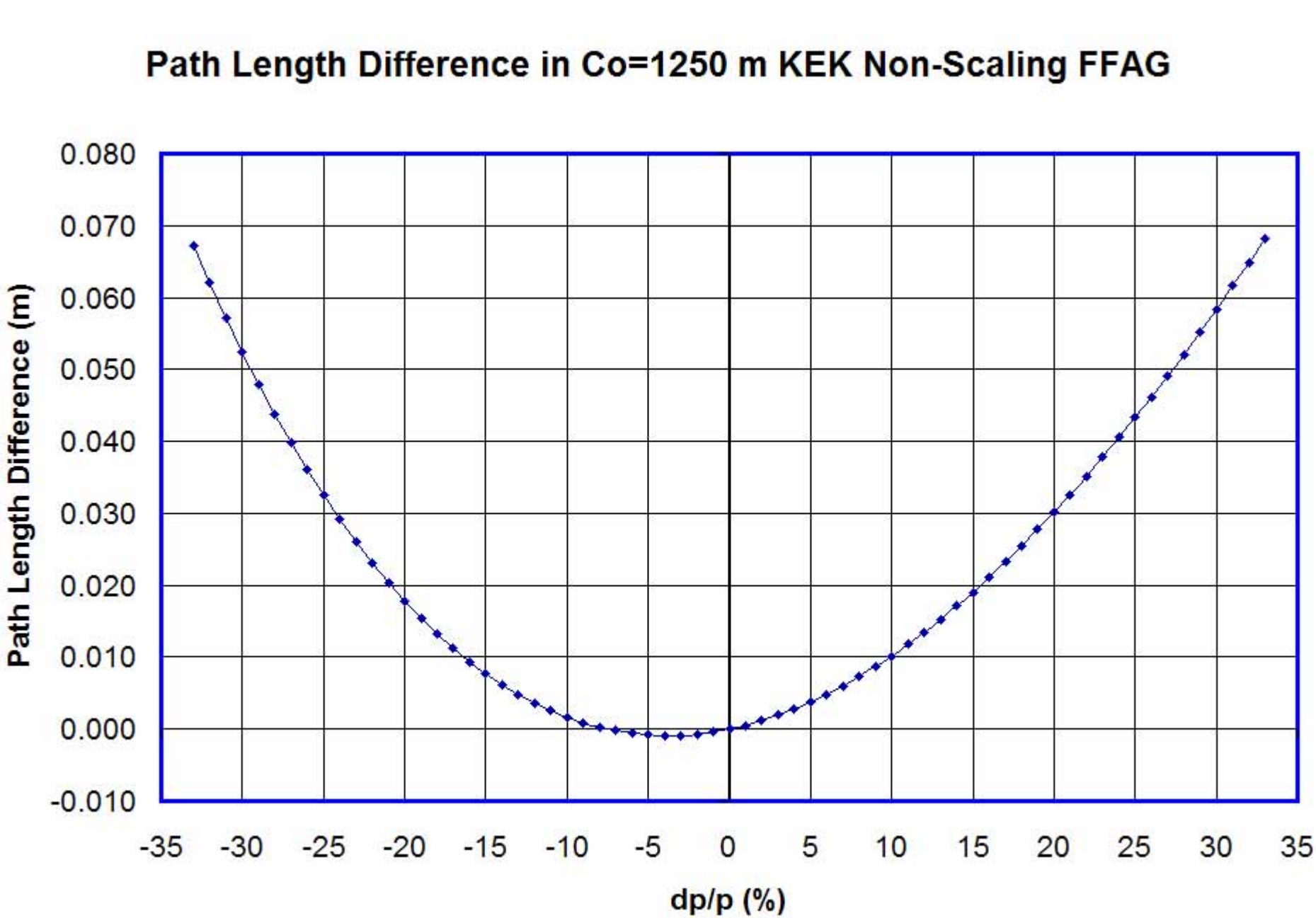
ANG(rad)	ANGQ(rad)	BYQ (T)	BY2 (T)
0.044213459	0.005995998	0.400000000	1.228971379

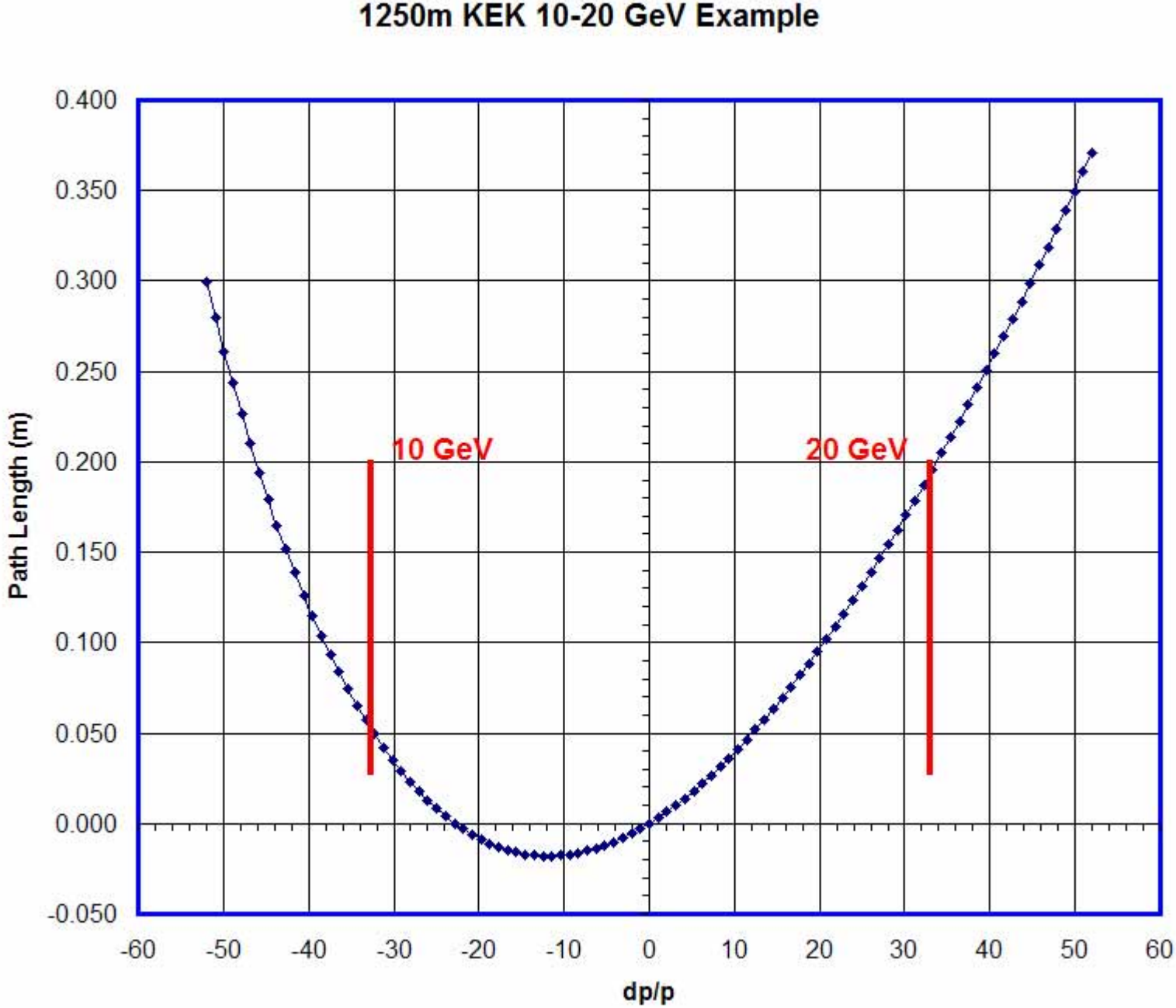

Betatron Tunes in the basic cell
Non-Scaling KEK 10-20 GeV C=1250 m
Betatron tunes
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
10
11
12
13
14
15
16
17
18
19
20
Momentum GeV/c
Horizontal tune
Vertical tune


Maximum Orbit Offsets in the KEK Non-Scaling FFAG lattice
Calculation by COSY
Maximum orbit offsets (m)
0.025
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
-0.020
10
11
12
13
14
15
16
17
18
19
20
Momentum (GeV/c)

PTC – Orbits at each momentum
1250 m




Path Length Difference in Co=1250 m KEK Non-Scaling FFAG
Path Length Difference (m)
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
-0.010
-35
-30
-25
-20
-15
-10
-5
0
5
10
15
20
25
30
35
dp/p (%)


1250m KEK 10-20 GeV Example
Path Length (m)
0.400
0.350
0.300
0.250
0.200
0.150
0.100
0.050
0.000
-0.050
-60
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
dp/p
10 GeV
20 GeV

Orbits at fixed negative γ_t – -50% - +50%
Large path length difference KEK 120 cells