

# FFAG and Weak Focusing Muon Gas-Filled 6-D Cooling Rings

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# CELL OF 4-CELL RING

$$\lambda = \rho / R_c$$

$$\sigma = (1 + \lambda^2 + 2\lambda \cos \theta)^{1/2}$$

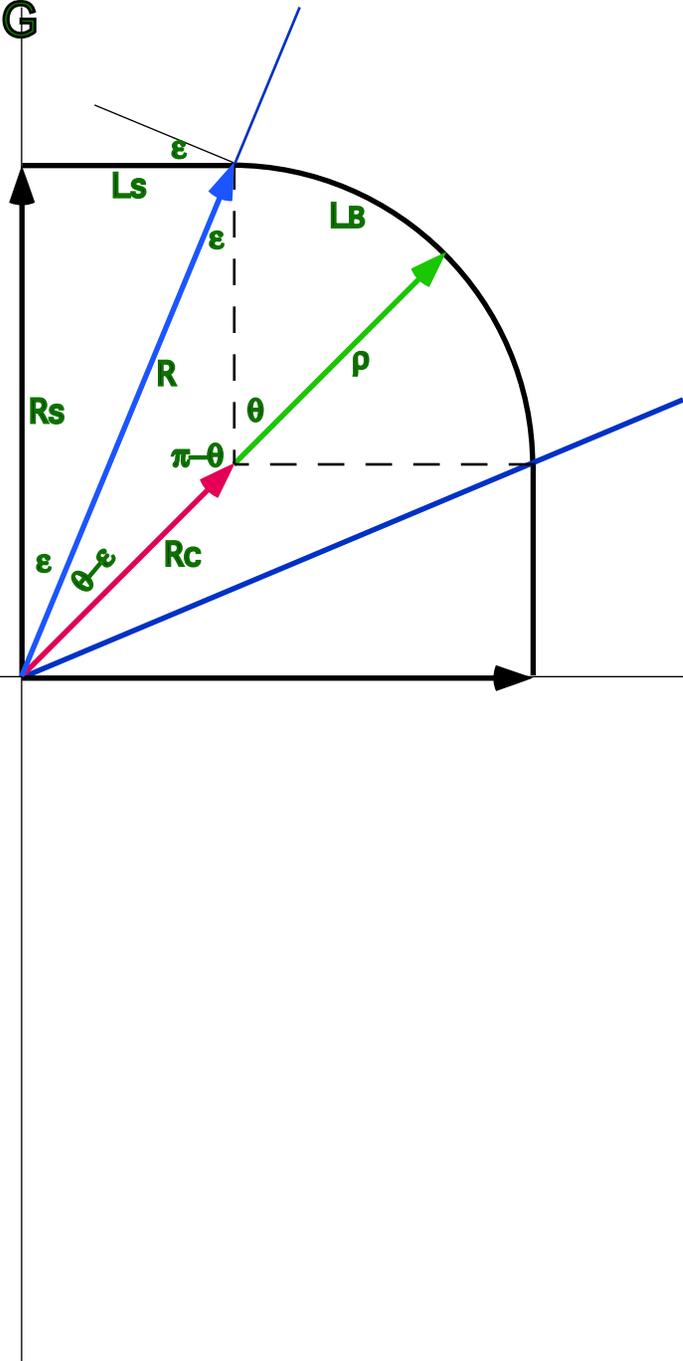
$$R = R_c \sigma$$

$$\varepsilon = \sin^{-1}(\sin \theta / \sigma)$$

$$R_s = R \cos \varepsilon$$

$$L_s = R \sin \varepsilon$$

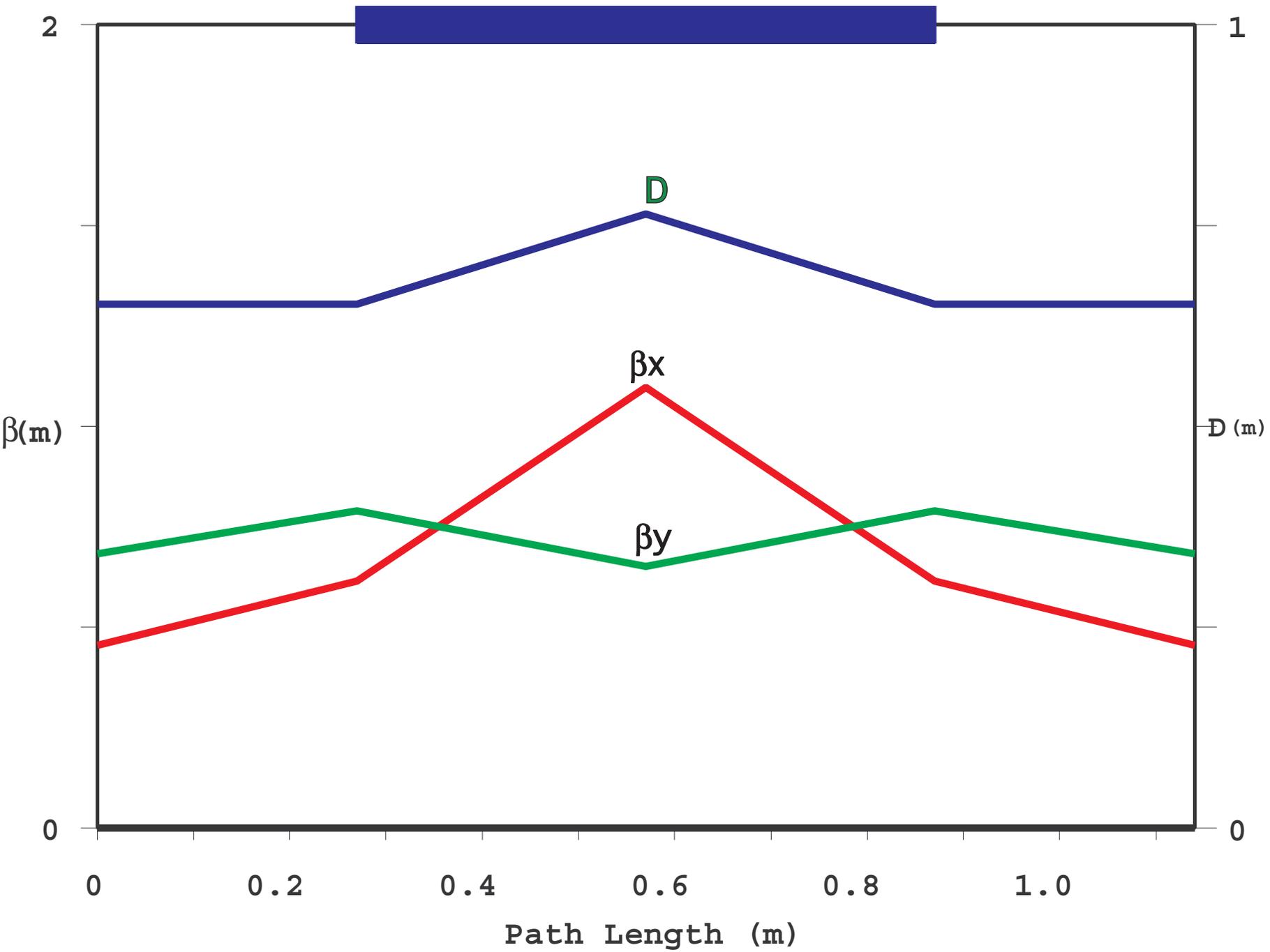
$$L_B = \rho \theta$$



# 4 DIPOLE RING

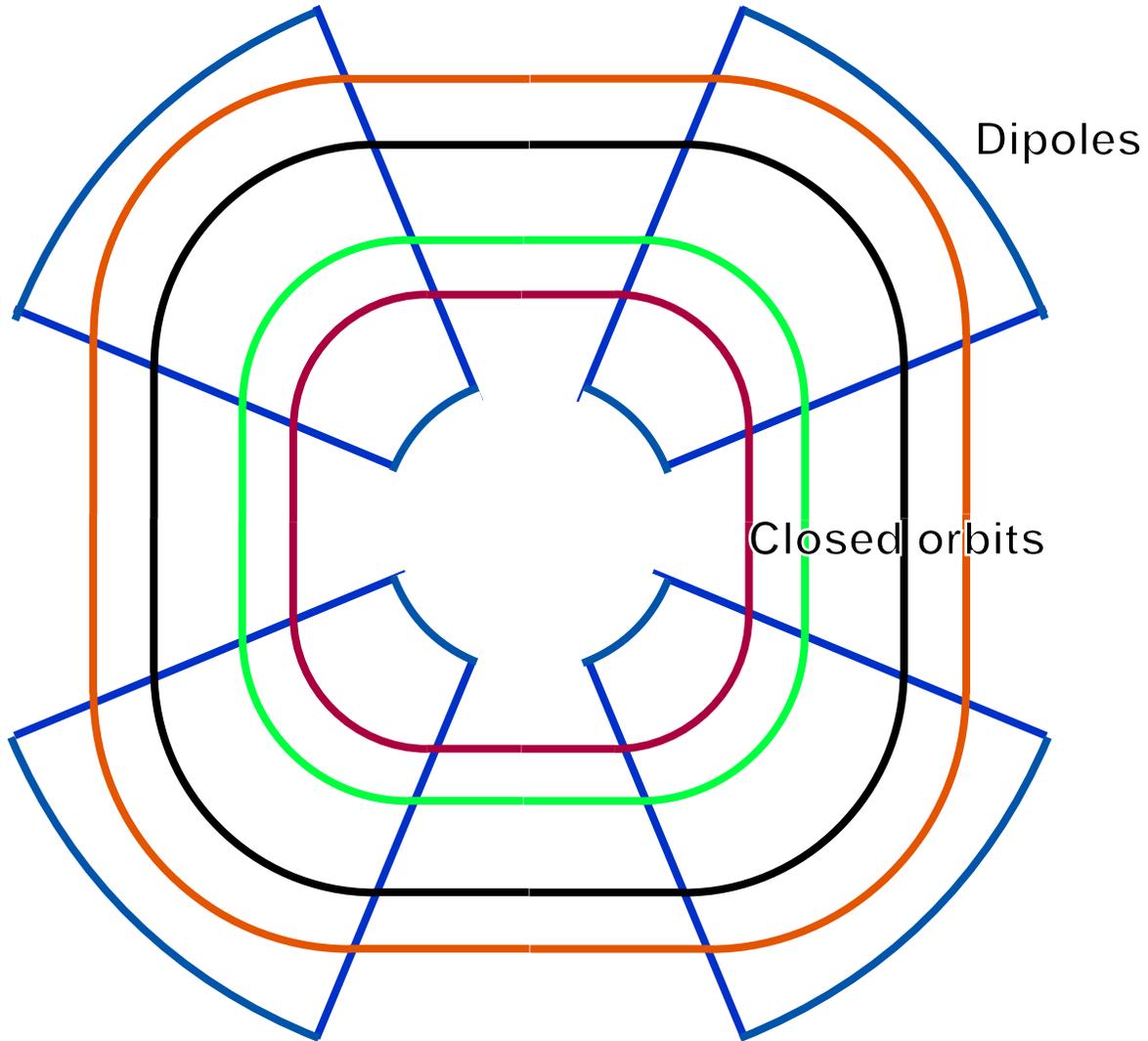
$$\lambda = \rho / Rc = 1$$

$$\rho = Rc = 0.382, LB = 0.25$$

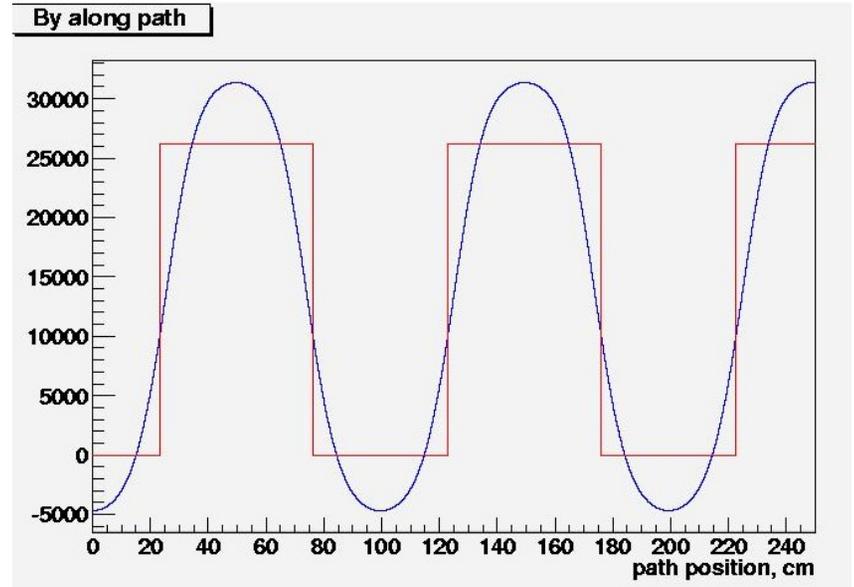
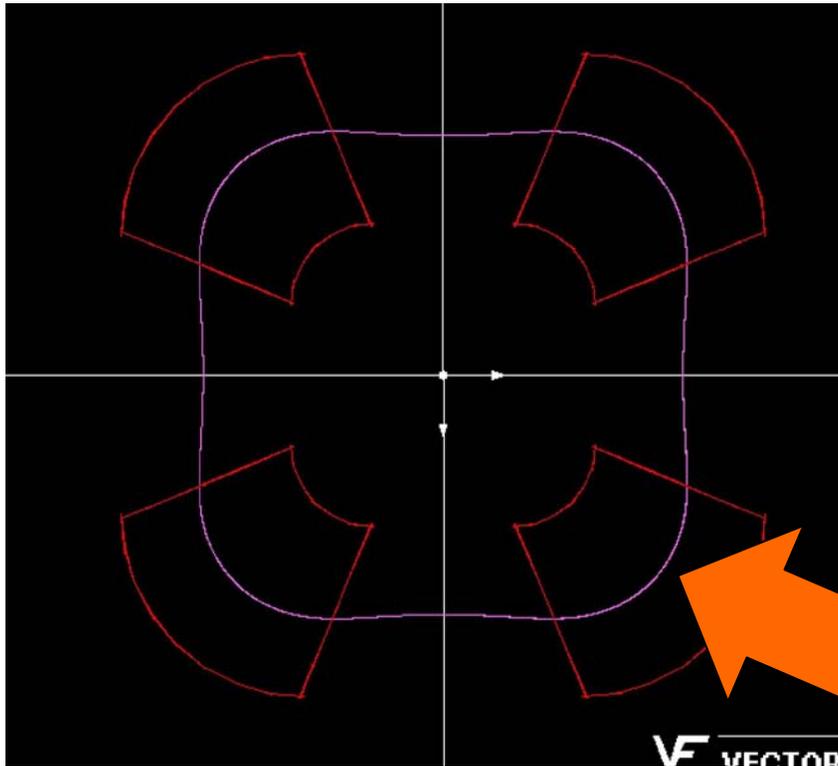


# 4-DIPOLE WEAK-FOCUSING RING

$$\lambda = \rho / R_c = 1$$



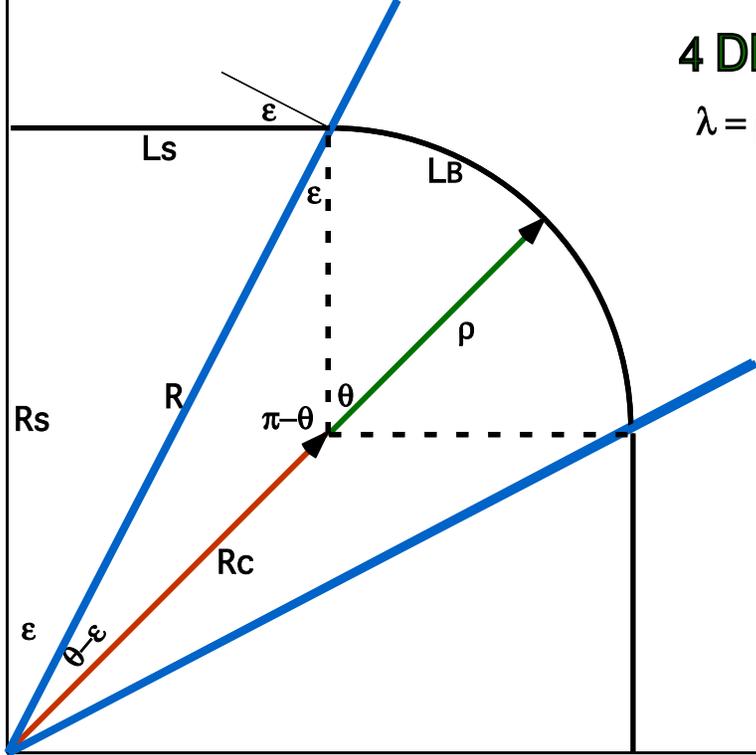
# Tosca Results – Steve Kahn



Closed orbit for  
250 MeV/c muons  
at  $r = 55.03\text{cm}$

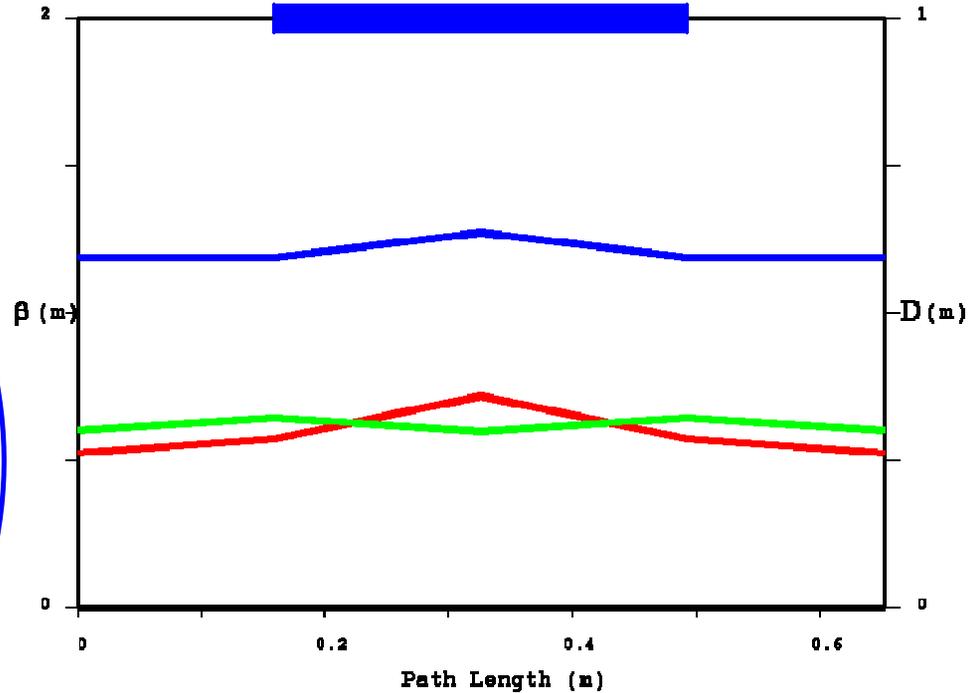
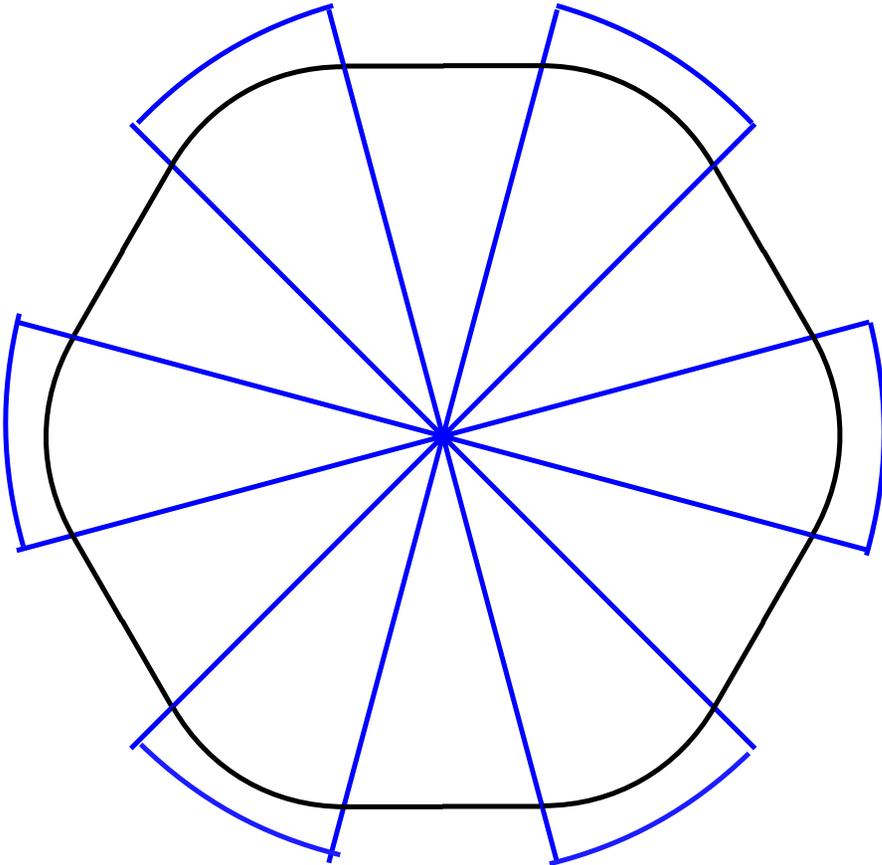
# 4 DIPOLE RING

$$\lambda = \rho / R_c = 2/3$$



# Gas Filled Dipole Wedge Rings

6 DIPOLE RING



Key parameters at  $r = 60$  cm

$\beta_x = 53$  to  $72$  cm ;  $\beta_y = 60$  to  $64$  cm

Dispersion =  $60$  to  $64$  cm

Circumference =  $3.91$  m

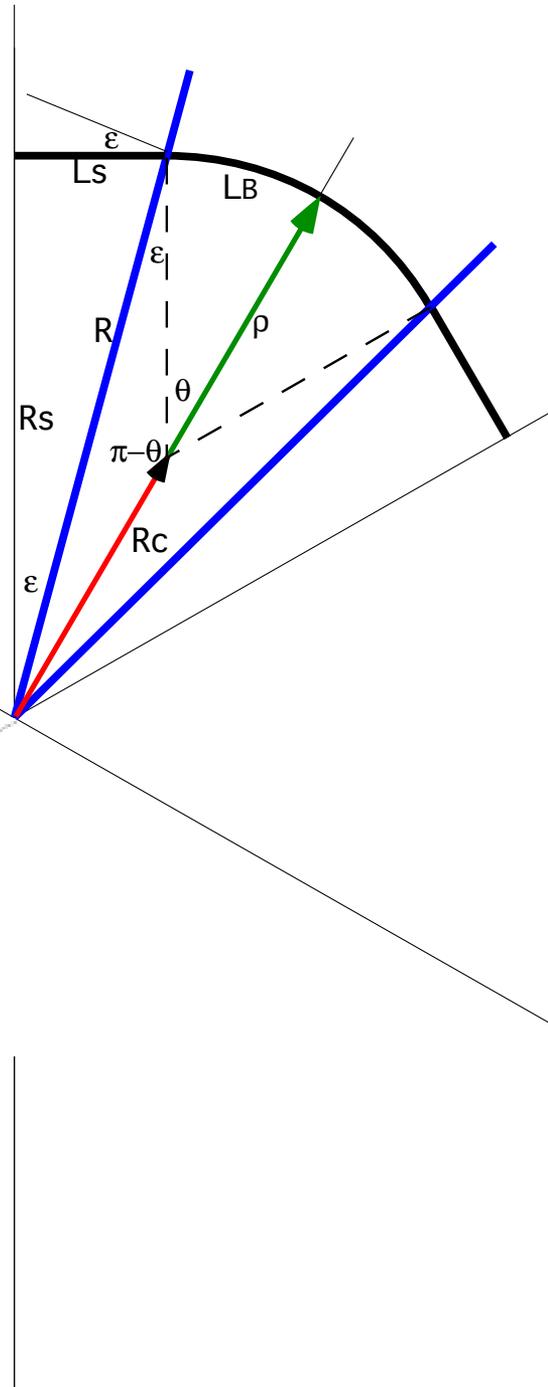
# 6 DIPOLE RING

$$\lambda = \rho / R_c = 1$$

Parameters of 6-Sector Rings

$\rho = R_c$ ,  $\theta = 30^\circ$ ,  $\epsilon = 15^\circ$

|                       |       |
|-----------------------|-------|
| Momentum, GeV/c       | 0.25  |
| Magnetic field, T     | 2.62  |
| Magnet half length LB | 0.167 |
| Gap half length LS    | 0.159 |
| Rho                   | 0.318 |
| Rc                    | 0.318 |
| Cell length           | 0.637 |
| Circumference         | 3.82  |
| RS                    | 0.594 |
| R                     | 0.615 |
| Bx, max               | 0.719 |
| By, max               | 0.645 |
| D, max                | 0.637 |



# Global Parameters Used

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Gas Density 100 atmospheres except where noted

Vertical Aperture +/- 15 cm

Horizontal Aperture +/- 25 cm

Gas filled RF cavities

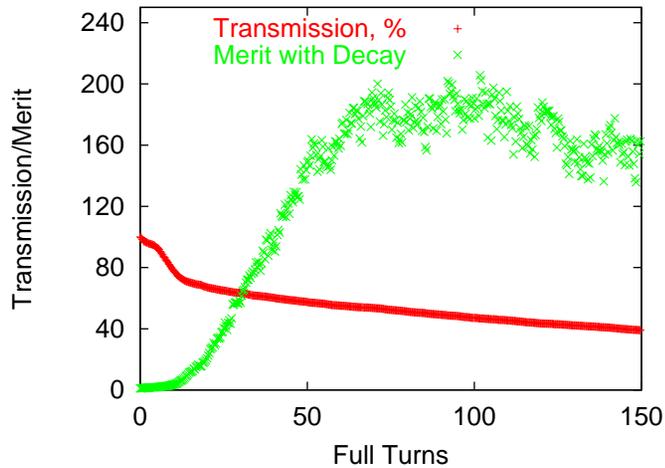
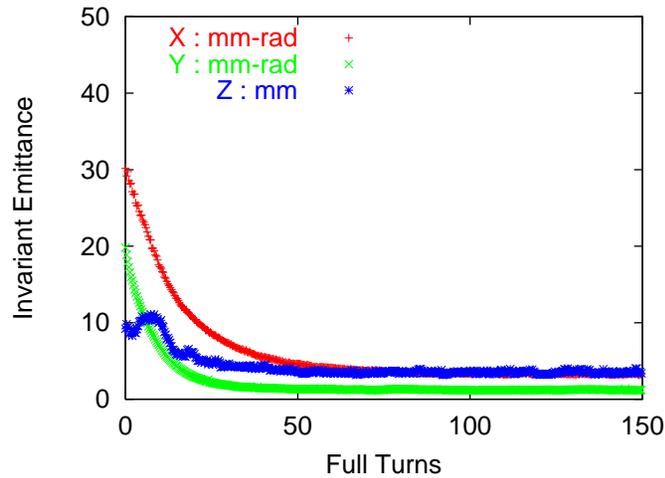
RF frequency 201.25 MHz

Hard edge magnetic fields

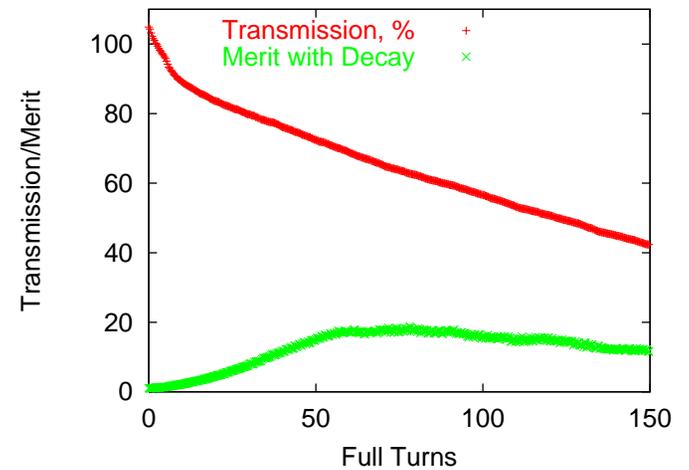
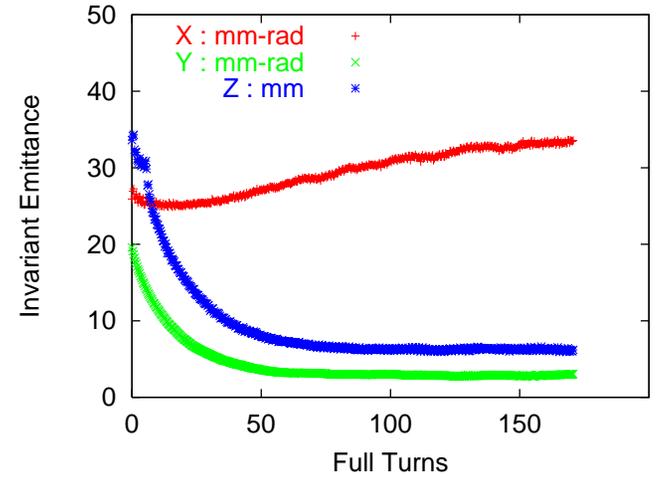
Except where noted ICOOL V2.66

# Recalculation with ICOOL V2.66

ICOOL V2.59

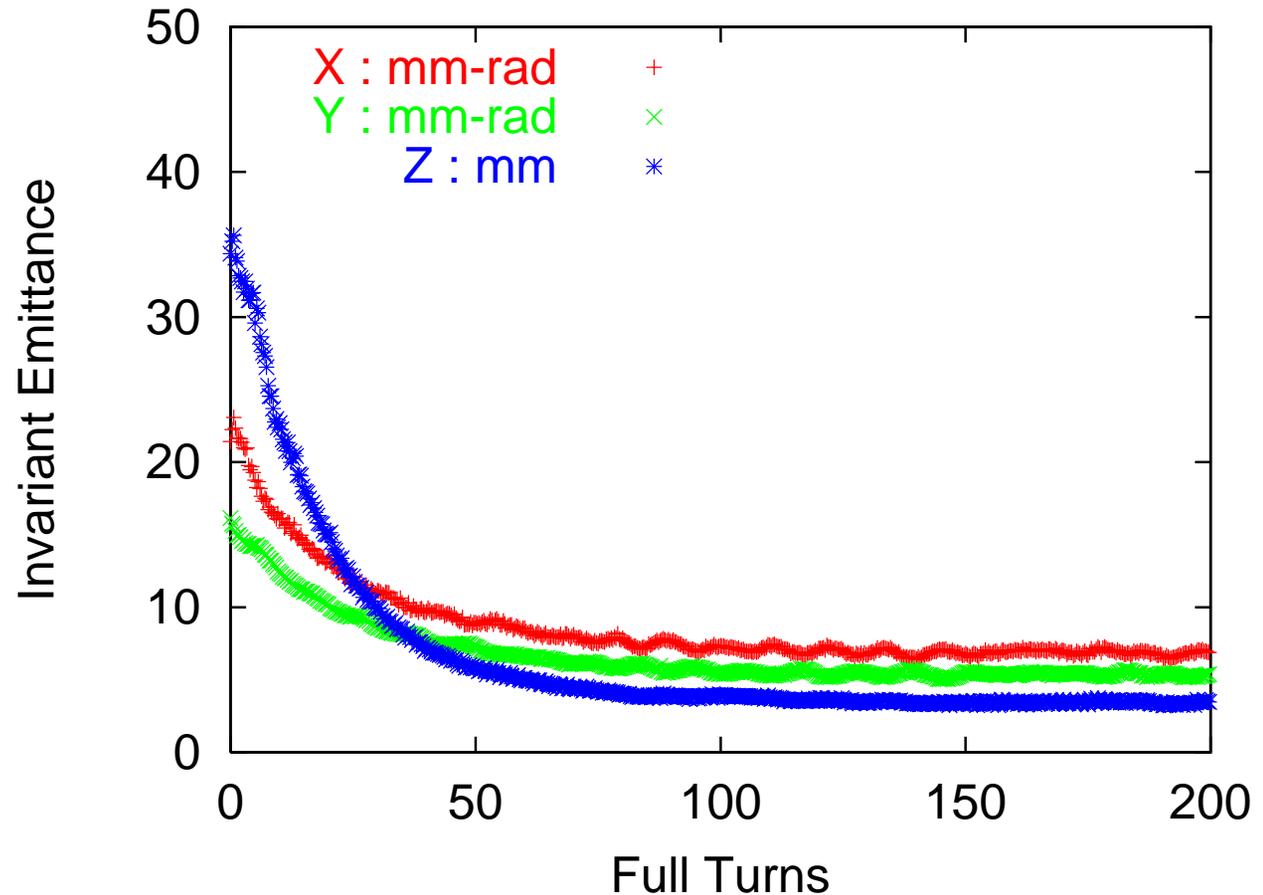


ICOOL V.266



# Introduce Skew Quadrupoles

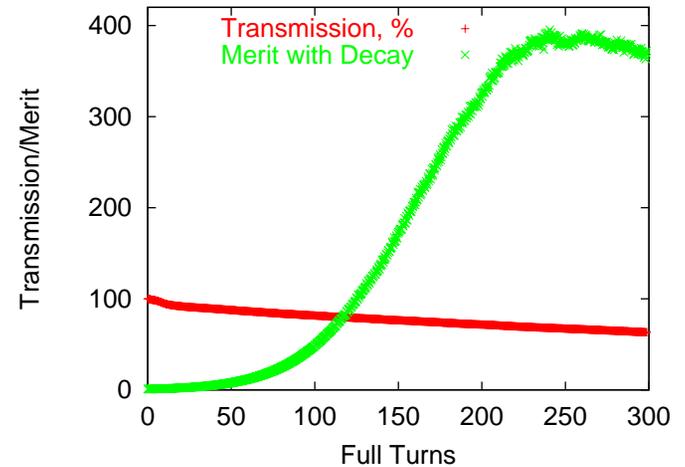
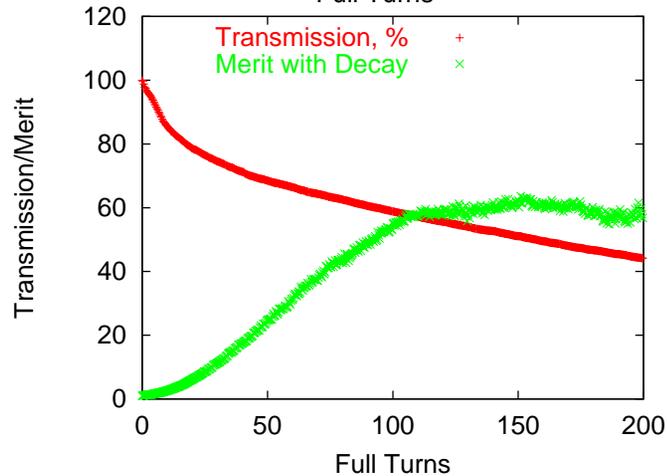
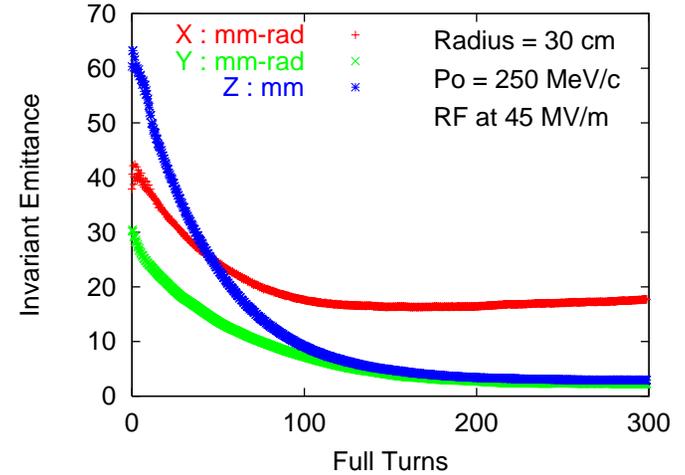
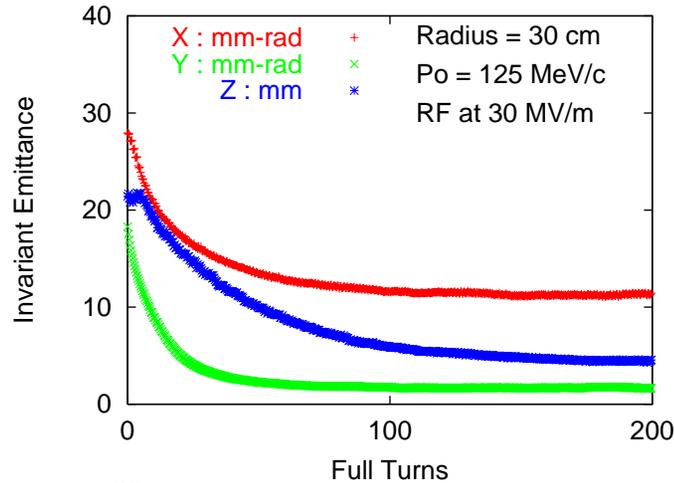
- Bracket dipoles with thin (3cm) skew quadroles
- Skew quadrupoles real estate at 9% circumference
- Test various gradients.
- X/Y Coupling achieved



# Reduced Radius Performance

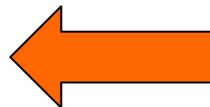
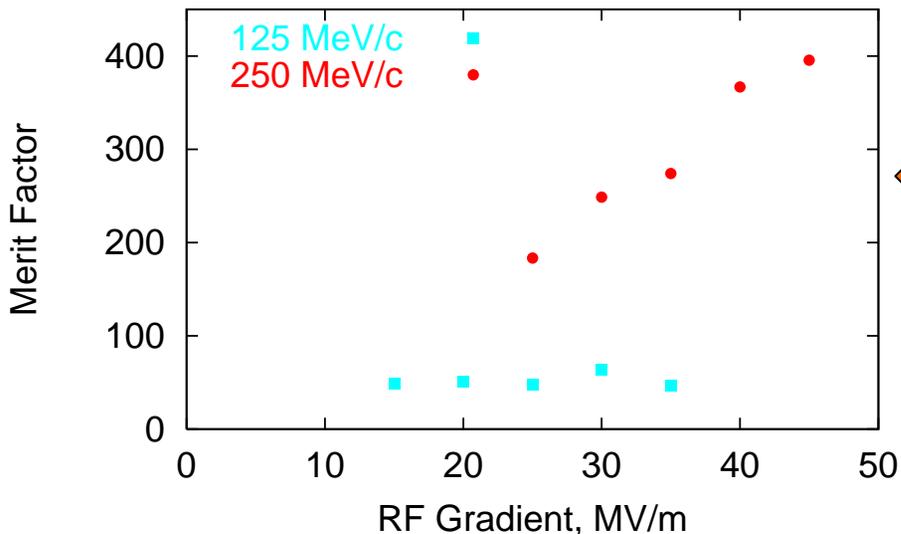
$B = 2.6T$   $P_o = 125$  MeV/c

$B = 5.2T$   $P_o = 250$  MeV/c



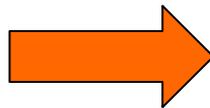
# Impact of RF on Performance

Reduced Radius Lattice

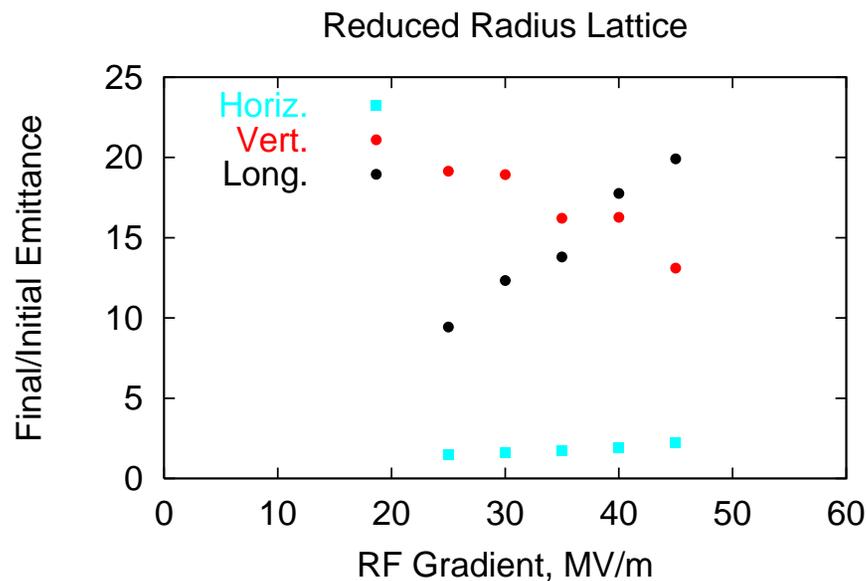


Gain in total merit factor results from increasing the rf gradient. Gain is only seen for the high-field,  $P_o=250$  MeV/c case.

High-field case. Gain comes from longitudinal cooling.



10 deg. Wedges



## **Strong Focusing Scaling Rings**

Following are two examples, both with 12 FFAG cells, one with and one without drifts.

### *Advantages*

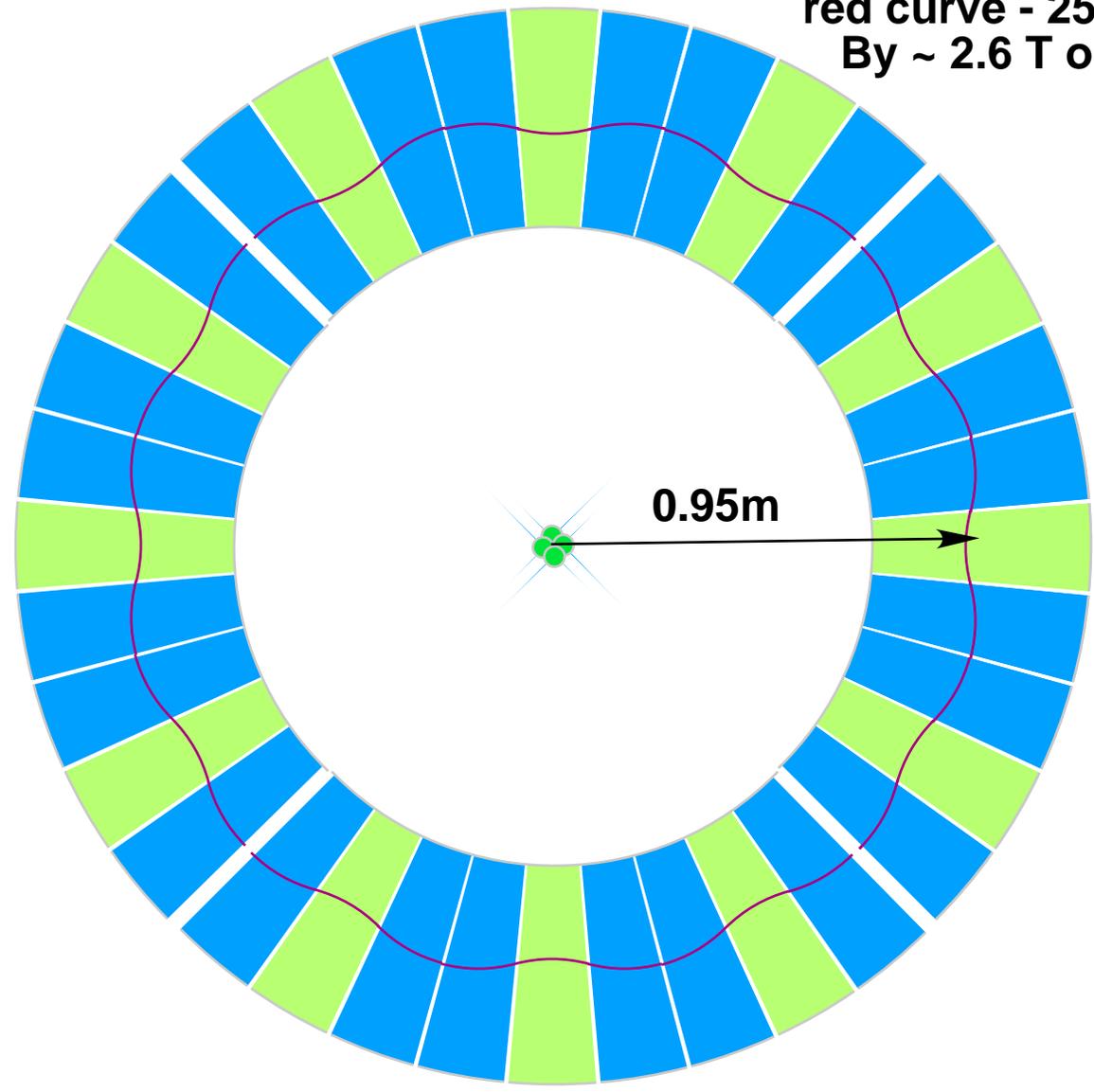
Allows lower magnetic fields and rf gradients

Large size > Accepts larger transverse beam size and longer bunch train

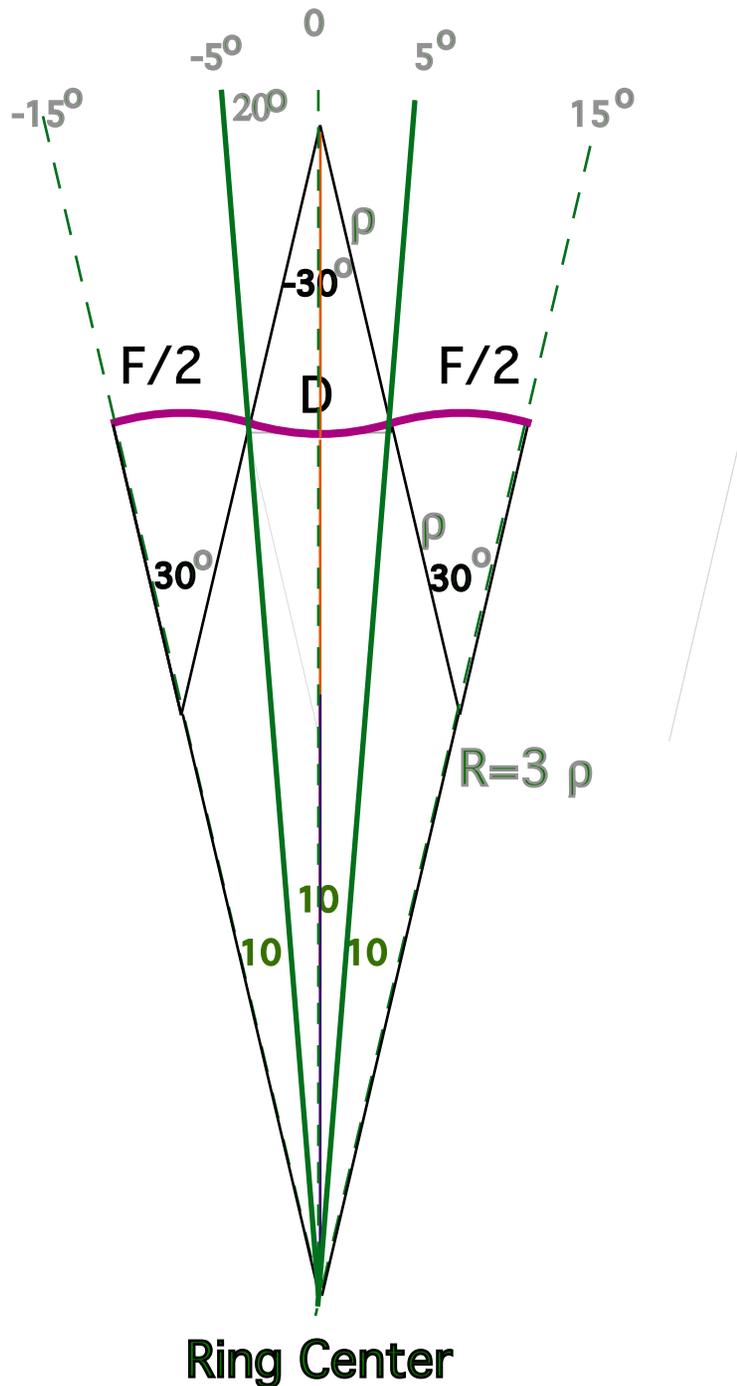
### *Disadvantage*

More complicated, higher cost

**12 CELL FFAG RING W/O DRIFTS**  
red curve - 250 MeV/c closed orbit  
By ~ 2.6 T on closed orbit



# 12 Cell Ring without Drifts Layout of 1 Cell



$$k = nd = -nf = 0.9$$

$$B/B_0 = (R/R_0)^k = (\rho/\rho_0)^{-n}$$

$$P/P_0 = (R/R_0)^{k+1}$$

$$(R/R_0) = (P/P_0)^{1/k+1}$$

$$D = dR/d(\rho/\rho_0) = R/(k+1) = .2581\text{m}$$

$$L_f = .3333\text{m} ; L_d = .1667\text{m}$$

$$L_{\text{cell}} = 0.5\text{m} ; \text{Circumference} = 6\text{m}$$

$$\rho_0 = .3183\text{m} ; R_0 = .9549\text{m} ; B_0 = 2.620\text{T}$$

$$B/B_0 = (R/R_0)^k = (\rho/\rho_0)^{-n}$$

$$P/P_0 = (R/R_0)^{k+1}$$

$$(R/R_0) = (P/P_0)^{1/k+1}$$

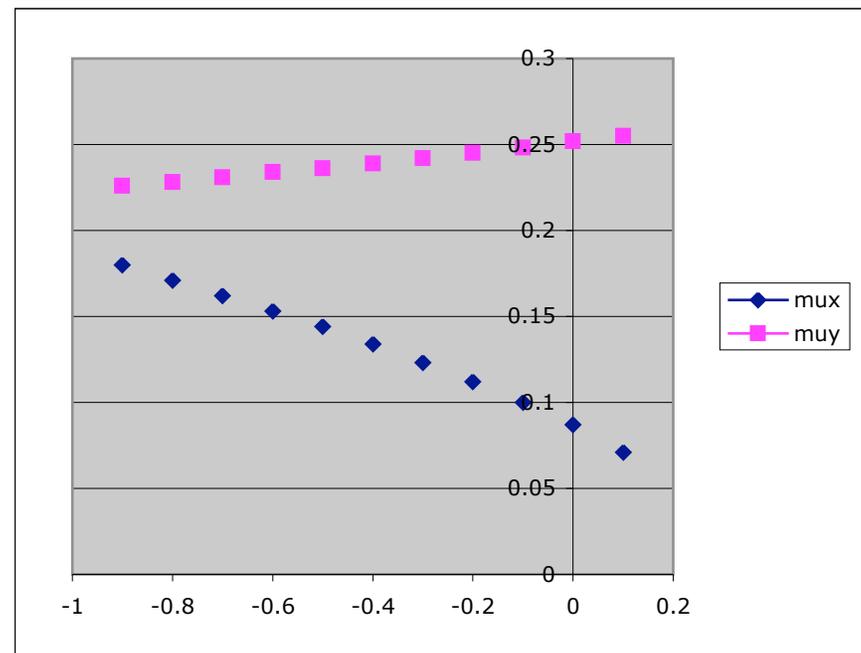
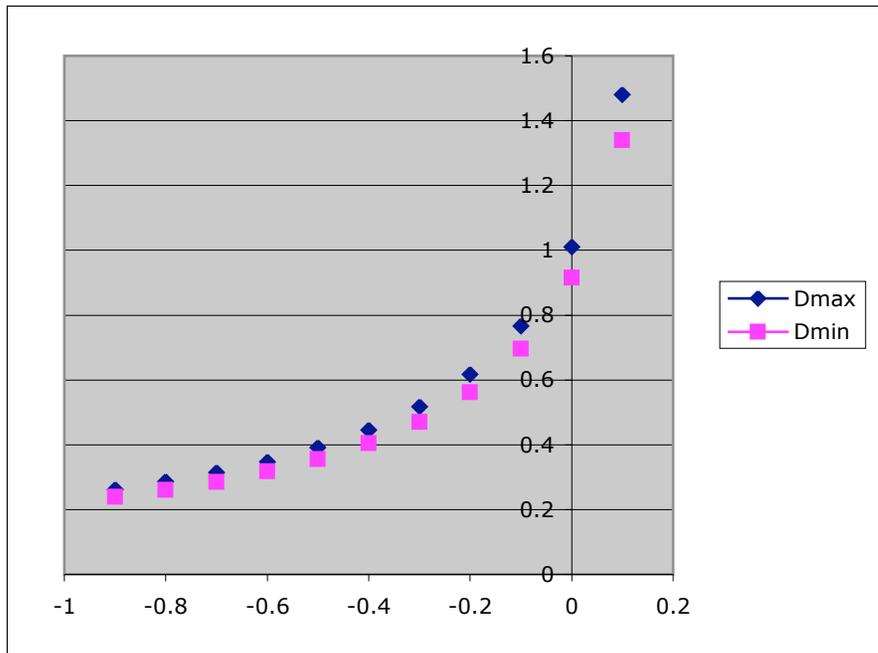
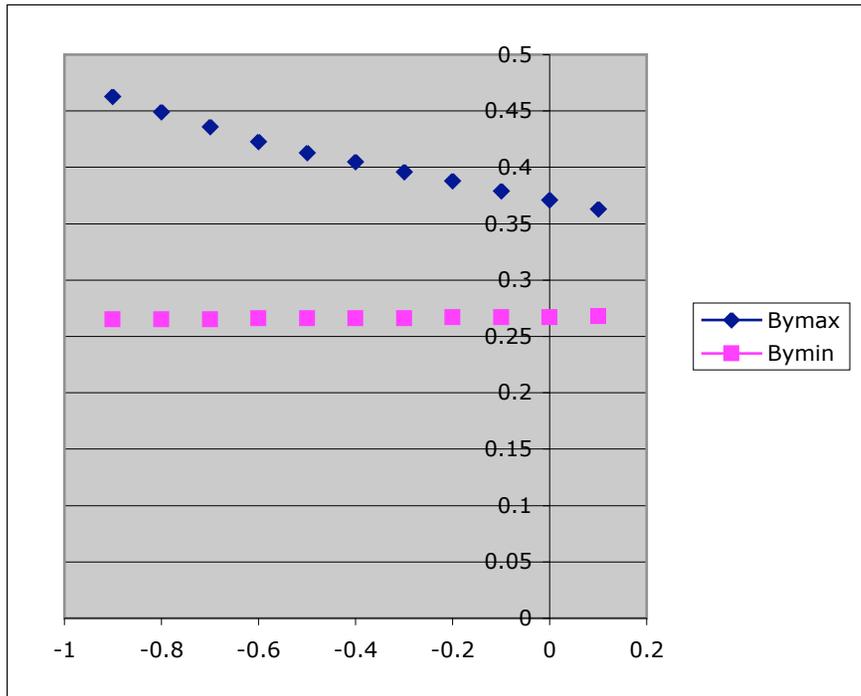
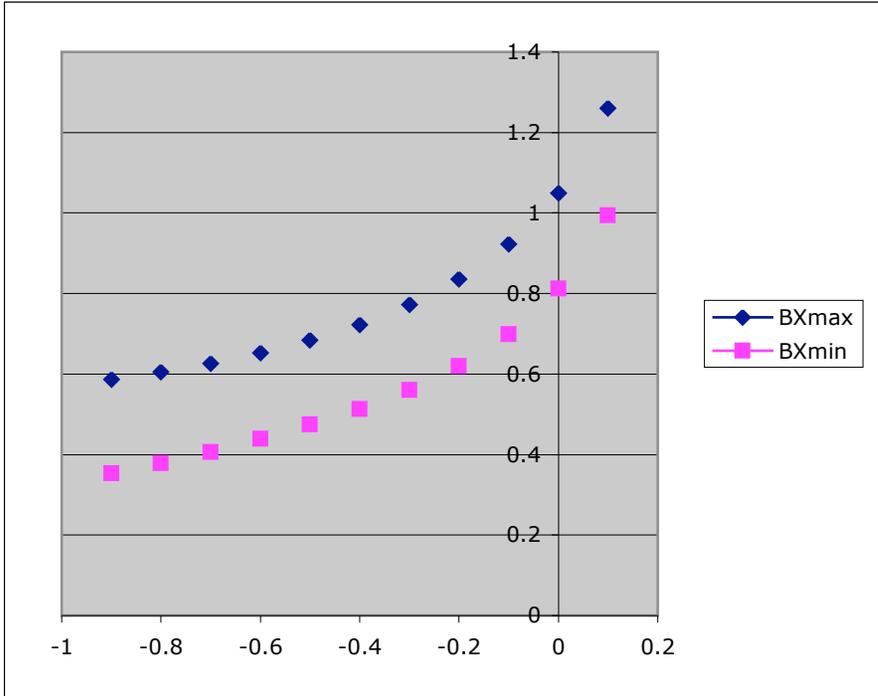
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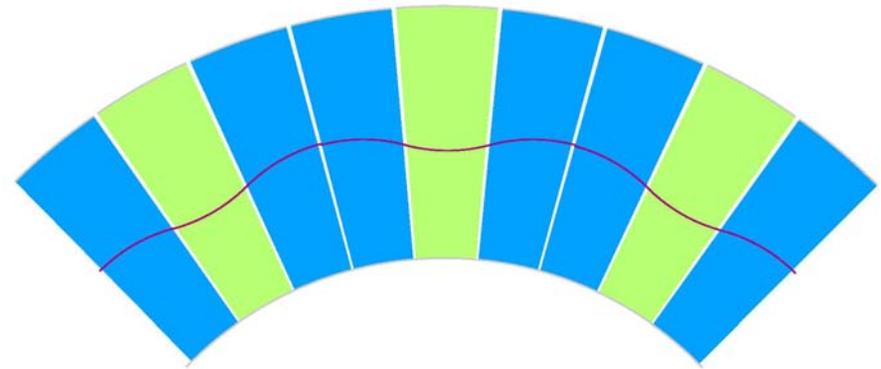
$$\rho_0 = .3183\text{m} ; R_0 = .9549\text{m} ; B_0 = 2.620\text{T}$$

# 12-CELL STRONG FOCUSING COOLING RING: ORBIT FUNCTIONS VS. FIELD INDEX $n$



# An FFAG-like Lattice

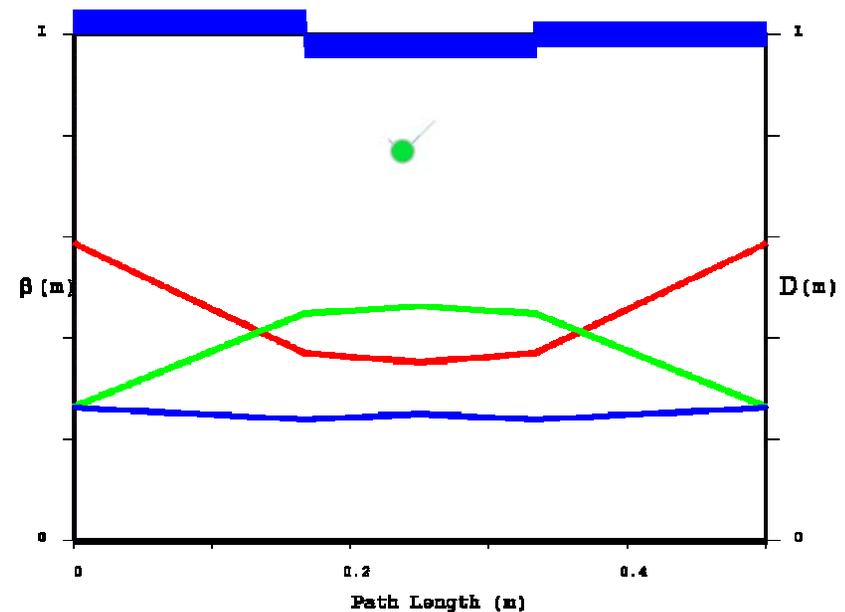
Lattice consists of alternating  
 Horz. Defocusing and Horz.  
 Focusing with  $L_{HD} = \frac{1}{2} L_{HF}$  .  
 No drift cells between dipole  
 elements.



3 Cells - 90°

## Parameters

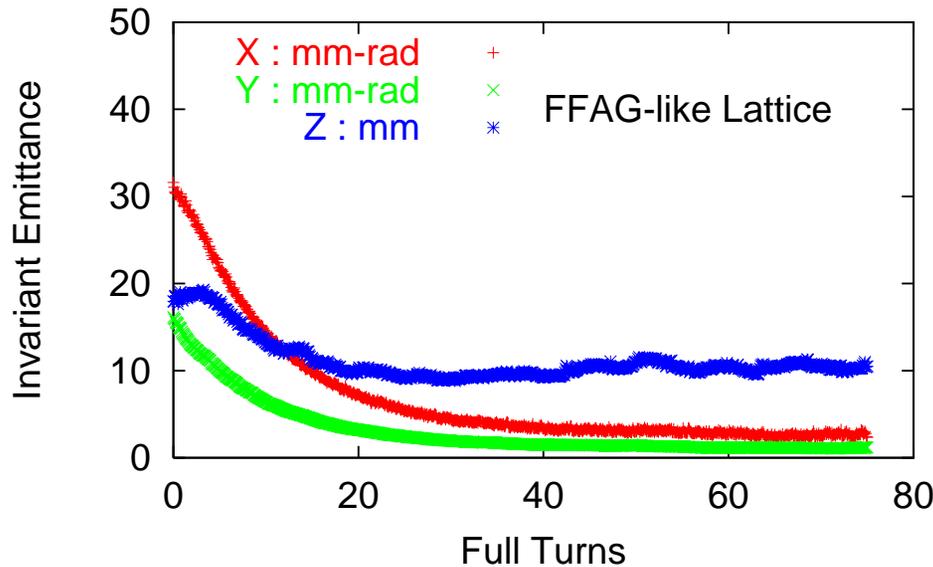
12 cells  
 Bend angles 30° and -15°  
 Circumference = 6m  
 $B_0 = 2.6T$  and  $P_0 = 250 \text{ MeV}/c$   
 Dispersion = 25 cm



| P/P0 | P<br>GeV/c | R/R0   | $\rho$<br>m | R<br>m | R-R0<br>m |
|------|------------|--------|-------------|--------|-----------|
| 1.3  | .325       | 1.0735 | .342        | 1.025  | .0702     |
| 1.2  | .300       | 1.0505 | .334        | 1.003  | .0482     |
| 1.1  | .275       | 1.0261 | .327        | 0.980  | .0249     |
| 1.0  | .250       | 1.0000 | .318        | 0.955  | .0000     |
| 0.9  | .225       | .9719  | .309        | 0.928  | -.0268    |
| 0.8  | .200       | .9415  | .300        | 0.899  | -.0559    |
| 0.7  | .175       | .9081  | .289        | 0.867  | -.0878    |

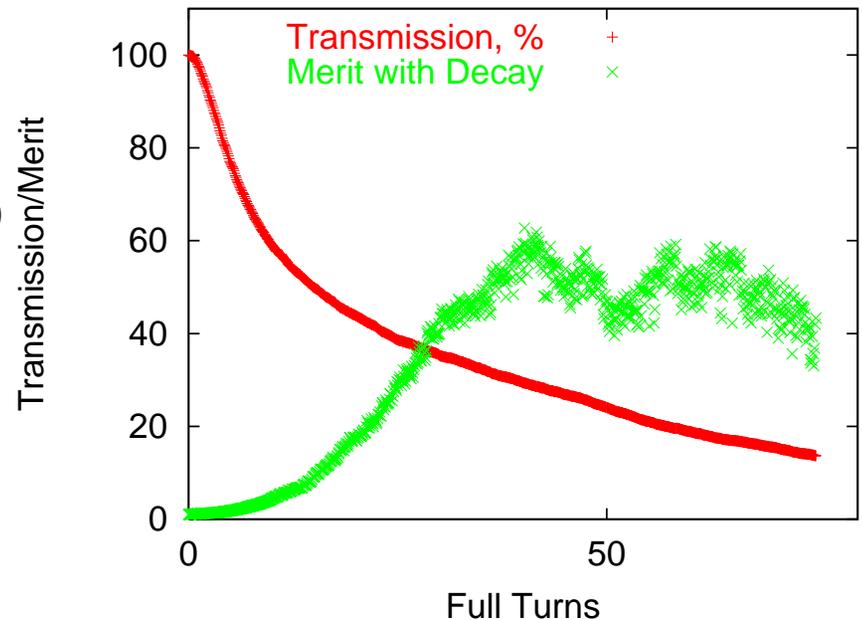
# FFAG Lattice Performance

12 Sector Oct. 21, 03 Lattice: 250 MeV/c



Horizontal Emittance Reduction Factor 10  
 Vertical Emittance Reduction Factor 11  
 Longitudinal Emittance Reduction Factor 2

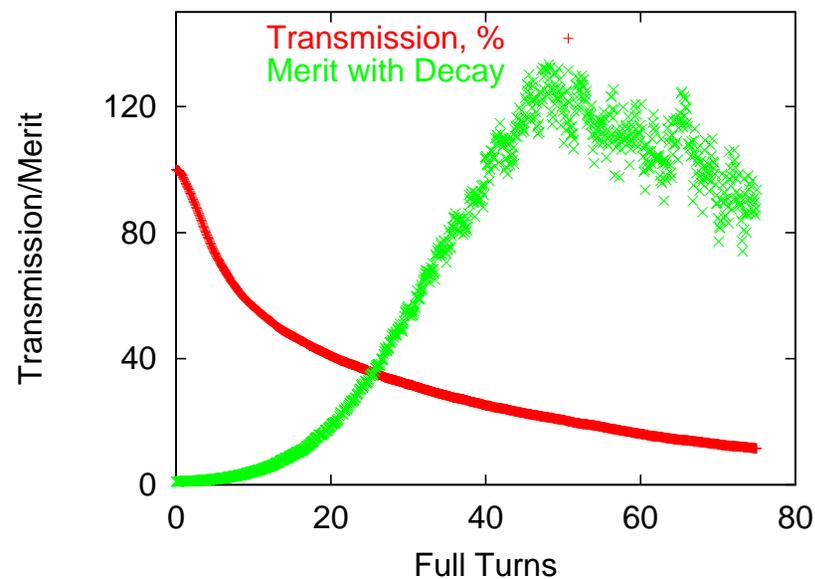
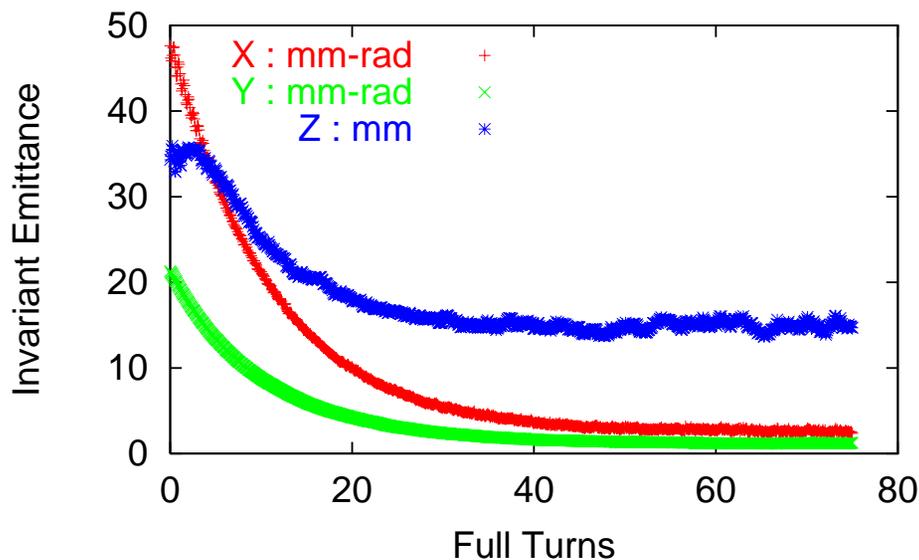
RF at 25 MV/m over  
 60% of circumference



# Strong Focusing Ring Performance

$$n = -0.6 \quad \text{rf at } 8 \text{ MV/m}$$

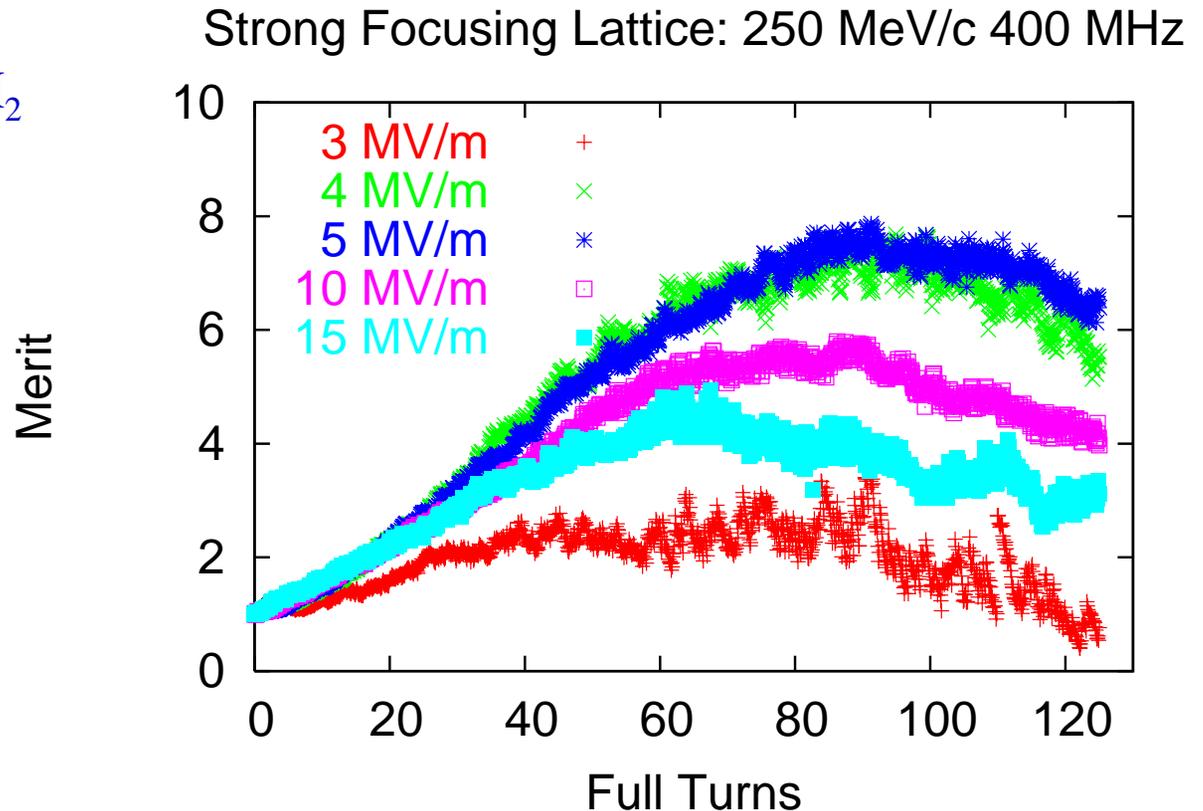
12 Sector Dec. 23, 03 Lattice: 250 MeV/c



# Strong Focusing Lattice

## Lattice Parameters

- RF at 400 MHz
- 40 Atmospheres  $\text{GH}_2$
- Aperture  $\pm 10$  cm
- Merit without decay
- $B_0 = 2.62$  T

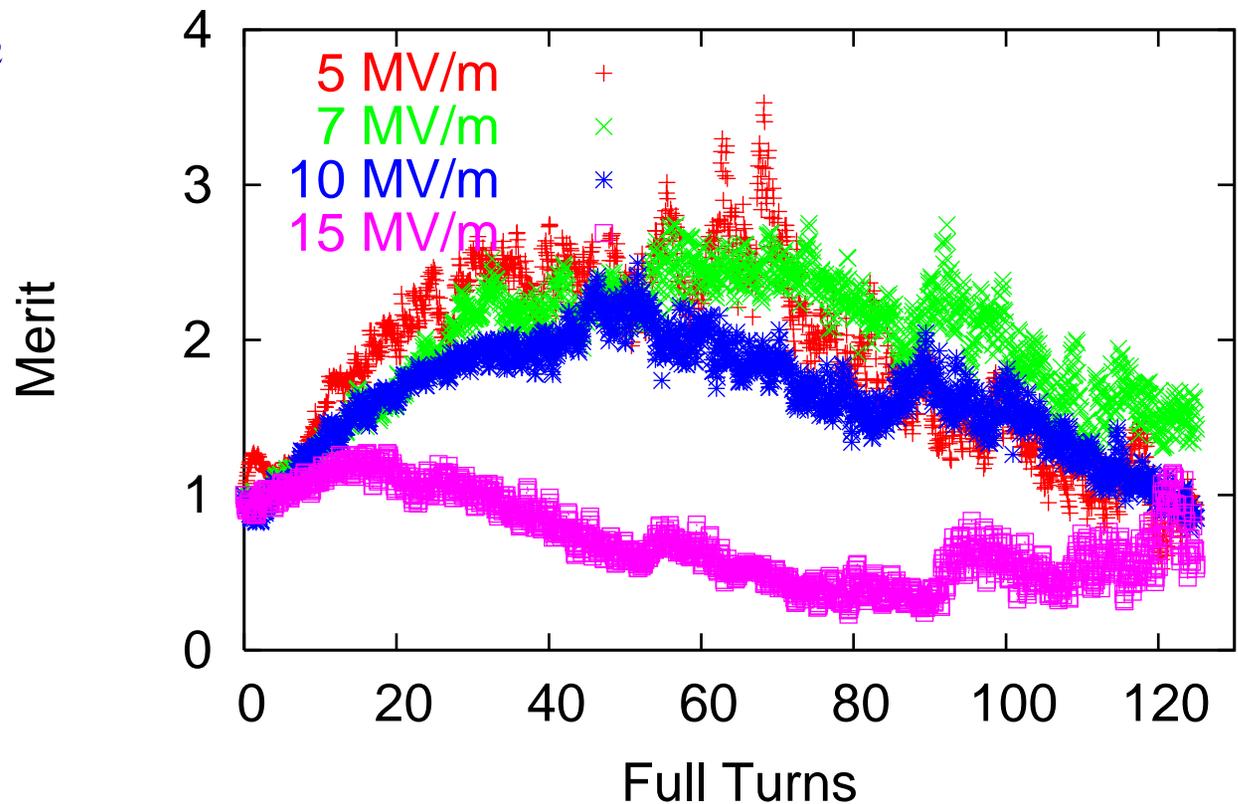


# Strong Focusing Lattice (cont)

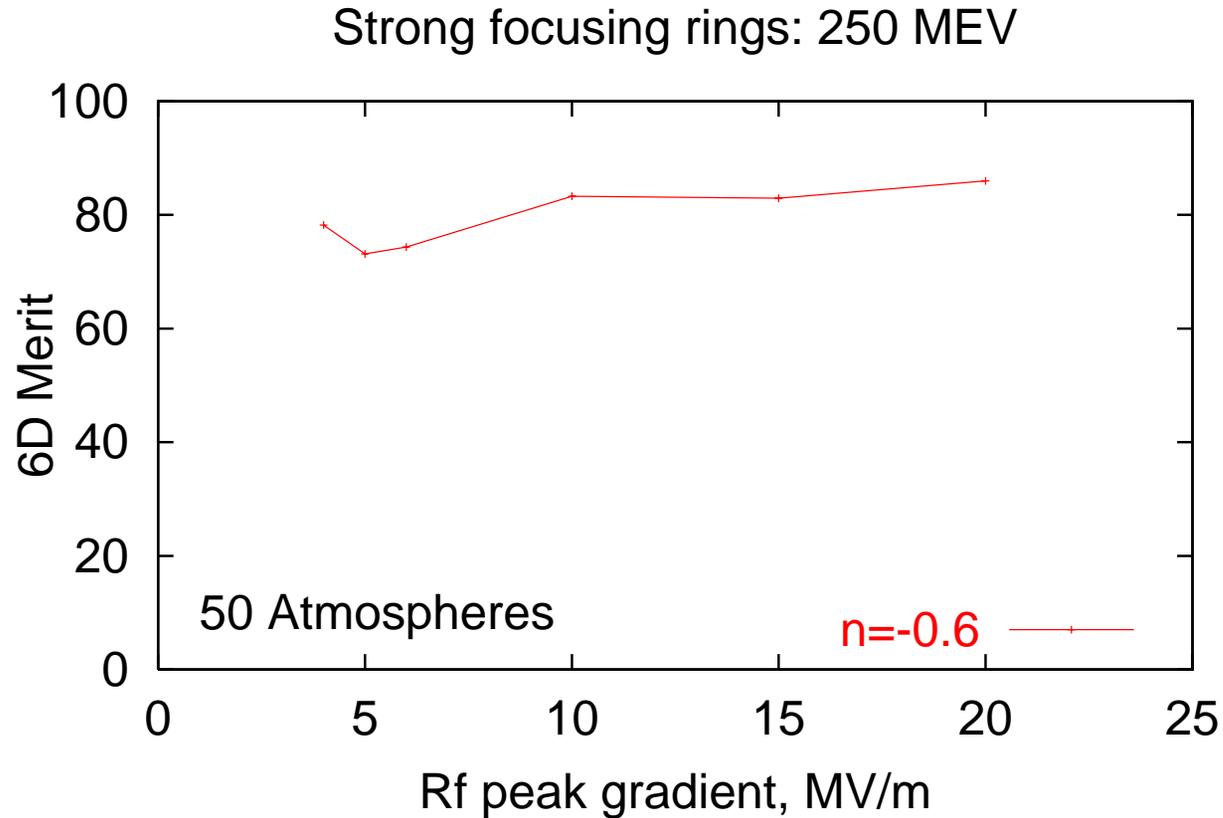
## Lattice Parameters

- RF at 800 MHz
- 40 Atmospheres  $\text{GH}_2$
- Aperture  $\pm 7.5$  cm
- Merit without decay
- $B_0 = 2.62$  T

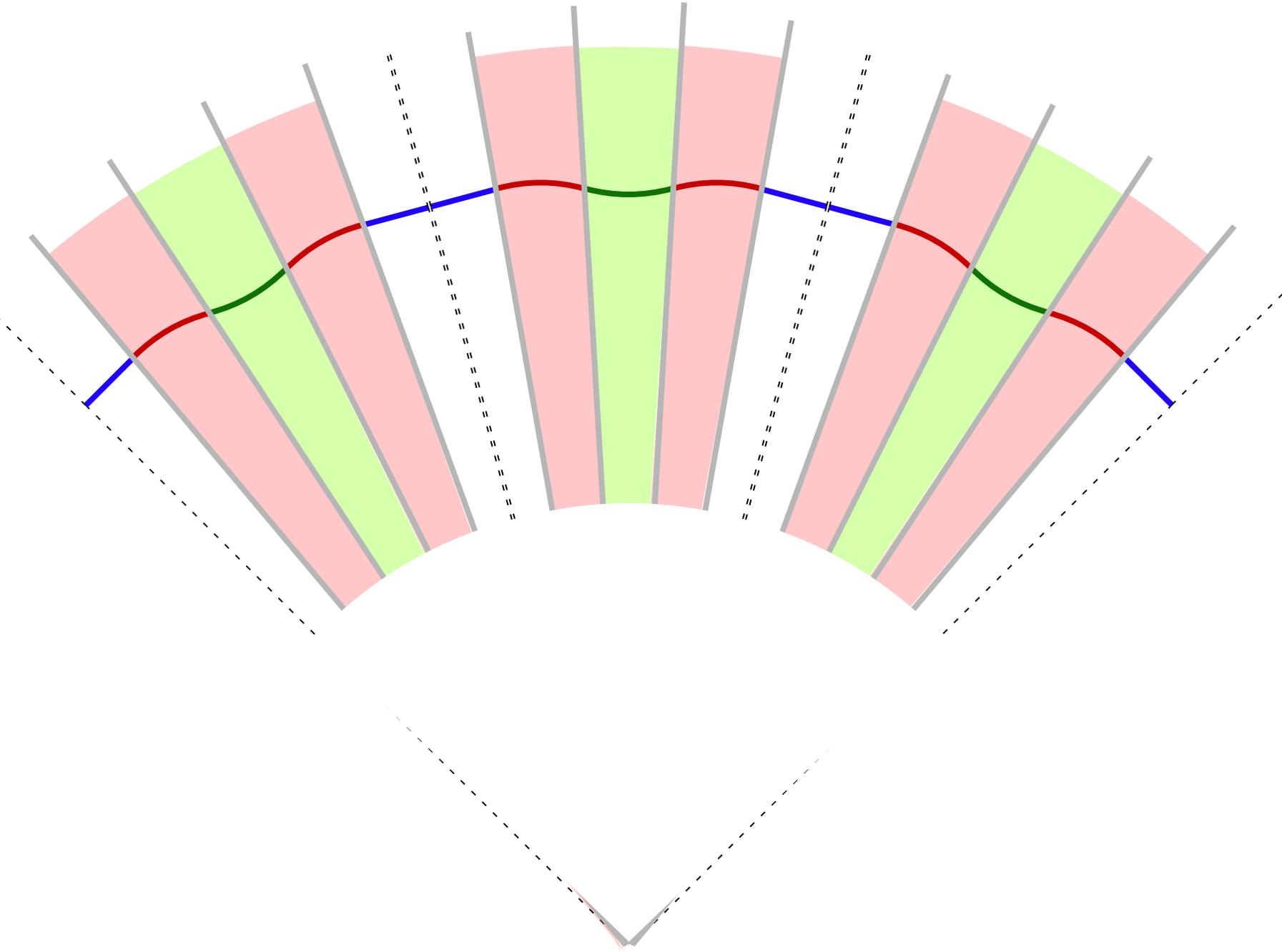
Strong Focusing Lattice: 250 MeV/c 800 MHz

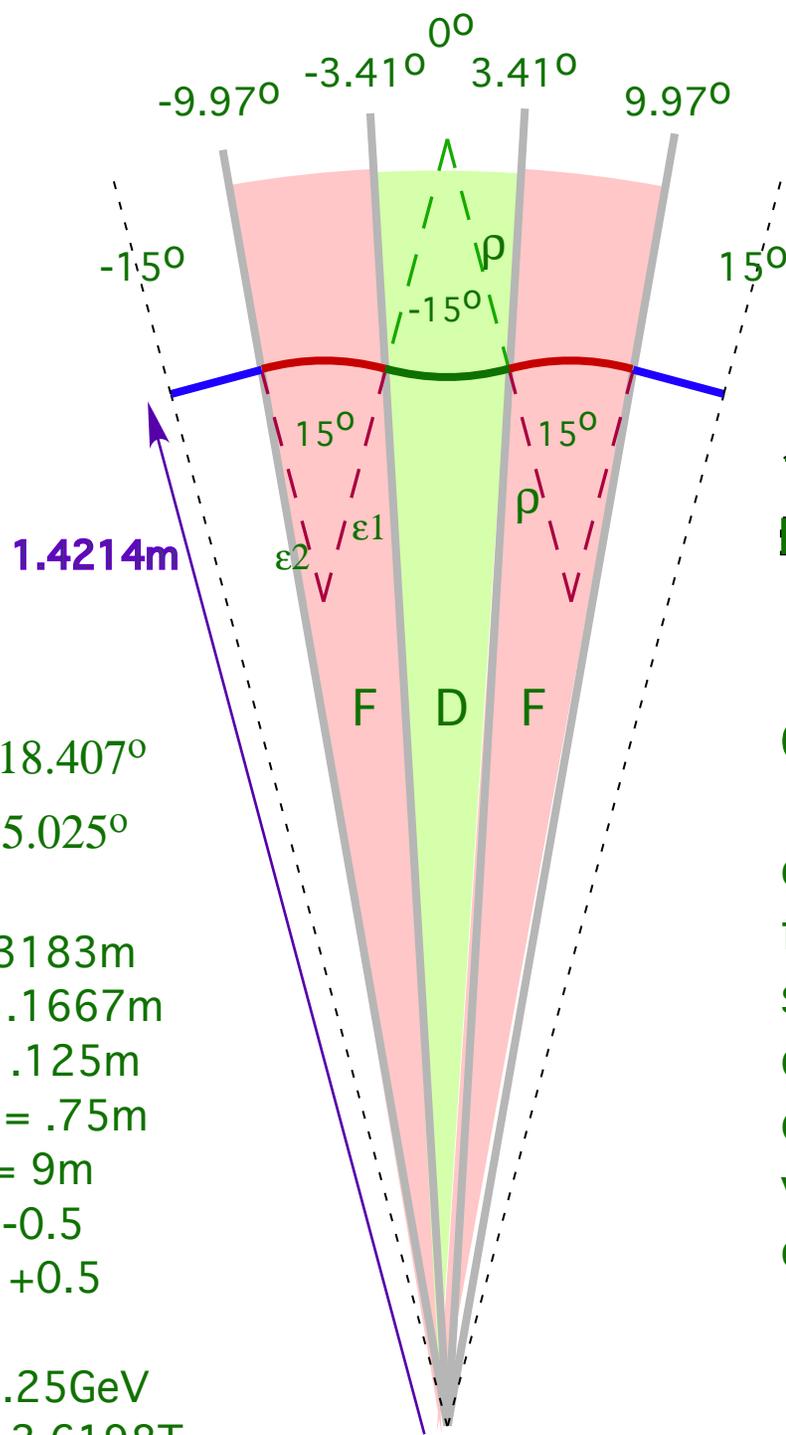


# 50 Atmospheres Performance



# 3 CELLS OF THE 12-CELL RING





## 12 CELL STRONG FOCUSING RING WITH DRIFTS

### Cell Diagram

The magnets have equal and opposite field strengths and field indices. The diagram shows the central, 0.25GeV closed orbit. Closed orbits of different momenta are scaled versions of the central closed orbit.

$$\epsilon_1 = 18.407^\circ$$

$$\epsilon_2 = 5.025^\circ$$

$$\rho = .3183\text{m}$$

$$\text{LB} = .1667\text{m}$$

$$\text{LO} = .125\text{m}$$

$$\text{Lcell} = .75\text{m}$$

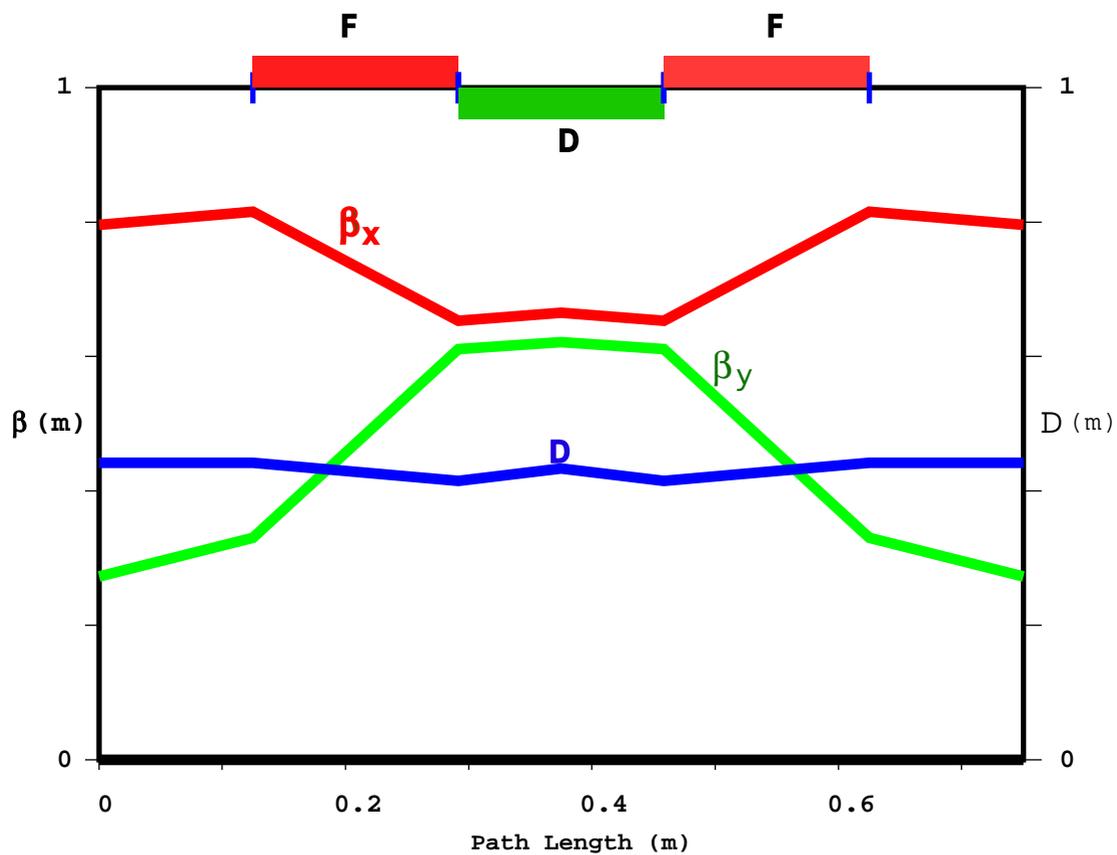
$$\text{Circ} = 9\text{m}$$

$$n_F \sim -0.5$$

$$n_D \sim +0.5$$

$$P_c = .25\text{GeV}$$

$$B_0 = 2.6198\text{T}$$



**Beta functions of one cell of FFAG 12-cell ring with drifts**

# Summary

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- The ICOOL fix ( $> V2.66$ ) significantly affected the performance of rings with gas loaded rf cavities
- The wedge dipole-only rings are still viable but with a reduction in scale or with the introduction of X/Y coupling.
- For weak-focusing lattices, high magnetic field , high rf gradients are favored.
- For strong-focusing lattices, low rf gradients are favored.

## Summary – continued

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- Weak focusing lattices are simpler and more compact than FFAG lattices and therefore are cheaper. The drift and magnet lengths are similar for best performance. Cooling is best vertically and longitudinally. For some cases skew-quadrupole correction is needed to cool horizontally.
- FFAG lattices give better performance at moderate fields than weak focusing lattices do. They have larger circumference and can contain a longer bunch train. They cool better transversely than longitudinally.
- The cell lengths of the two lattice types are roughly comparable to keep the beta functions low (0.5 – 1.0 m).
- A 200 Mhz RF system gives better cooling than 400 Mhz or 800 Mhz.