# A Magnetic Field Model for the Neutrino Factory FFAG Accelerator

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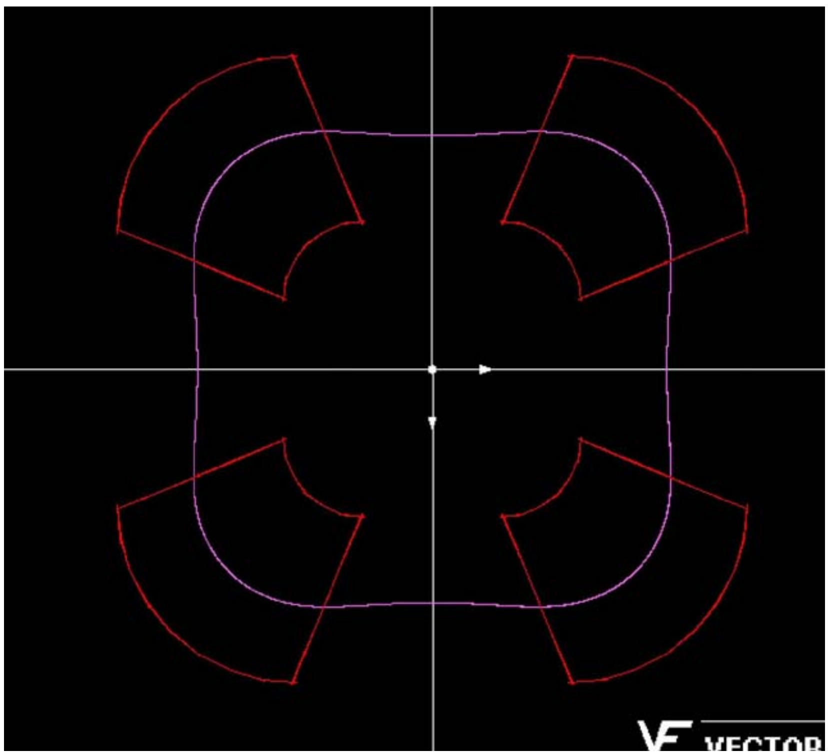
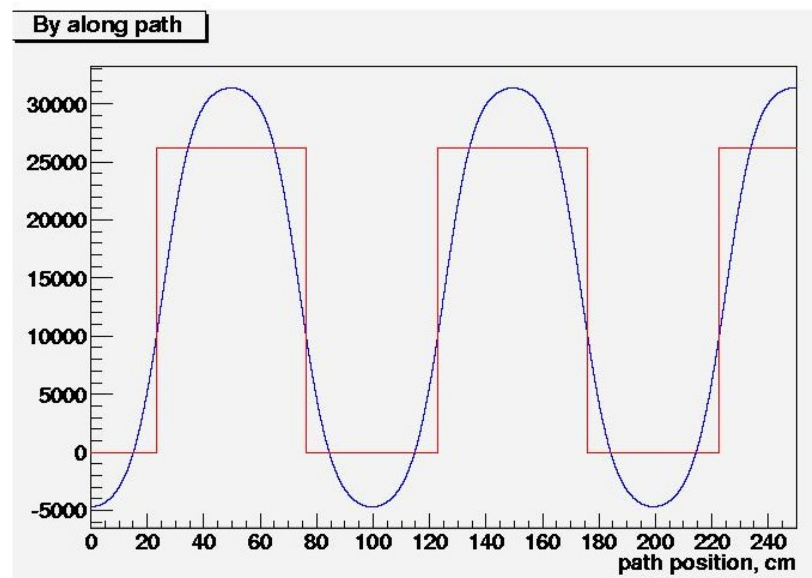
# Purpose of Study

- We would like to provide a model of a FFAG Triplet that could be used for the muon accelerator in a neutrino factory.
- We would like to generate a realistic field for this system that would include ends.
  - No hard-edge or other approximations that violate Maxwell's equations.
  - We would like to be able to use this realistic field map for tracking.
    - Specifically we would like to provide the field description for tracking with ICOOL. ICOOL only knows about a local coordinate system about some reference path.
- We would examine the dynamic aperture of this FFAG accelerator.

# Weak Focusing Muon Cooling Ring as an Example




- Use TOSCA to provide field map and closed orbit reference path.
- Provide field as Fourier series for use by Icool for tracking.



# Storage Ring Parameters

Muon Cooling Ring

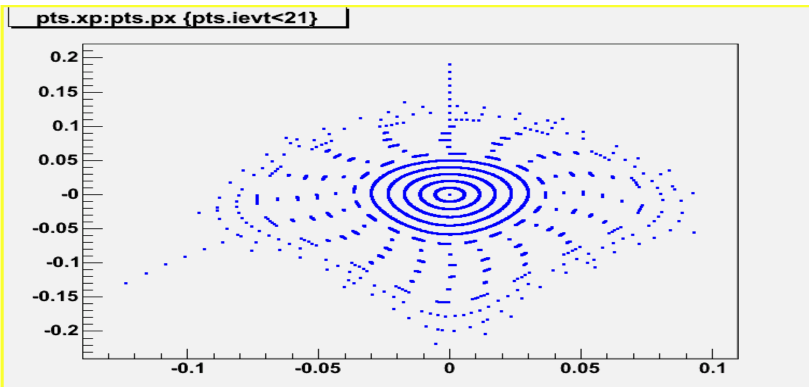
- The table below shows the Twiss Parameters as seen in ICOOL for both the *realistic* and *hardedge* models. These were calculated in a manner similar to those shown before
- Both ICOOL models look reasonably comparable to the original SYNCH and TOSCA models.
  - This is extremely encouraging and says that the realistic fields do not significantly alter the lattice!

| Parameter | A. Garren Synch | Tosca | Icool Realistic | Icool Hardedge | Icool with No Sex |
|---|---|---|---|---|---|
| $\mu_x$ | 99.8784° | 98.38° | 105.496° | 103.626° | 106.313 |
| $\beta_x$ | 37.854 cm | 32.3099 cm | 34.293 cm | 38.8635 cm | 33.6023 cm |
| $\alpha_x$ | 0 | -0.00124 | -0.000461 | -0.000576 | -0.00593 |
| $\mu_y$ | 92.628° | 100.62° | 100.619° | 94.9662° | 100.865 |
| $\beta_y$ | 56.891 cm | 53.9188 cm | 54.086 cm | 56.9616 cm | 53.844 cm |
| $\alpha_y$ | 0 | 0.0009894 | 0.000652 | -0.000001 | 0.00597 |

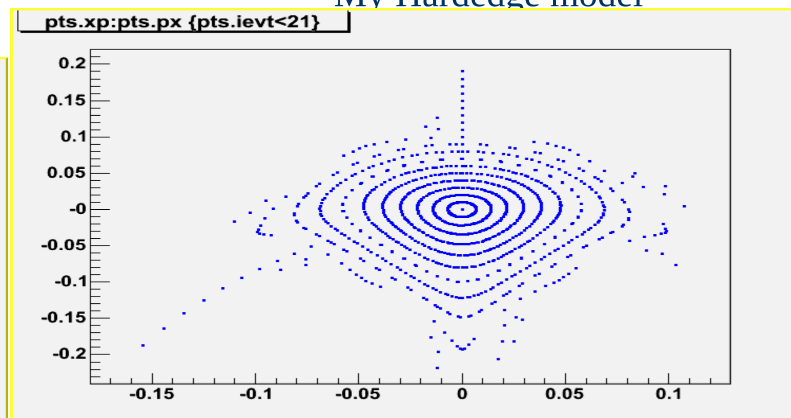
# Horizontal Dynamic Aperture ($x$ vs. $p_x$)

Muon Cooling Ring

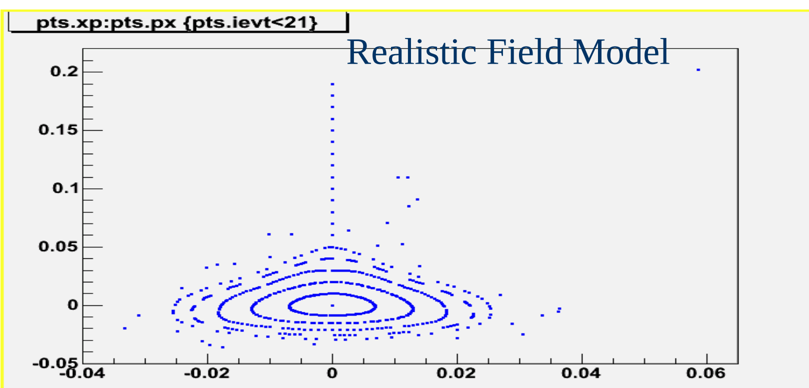
Kirk's Hardedge model



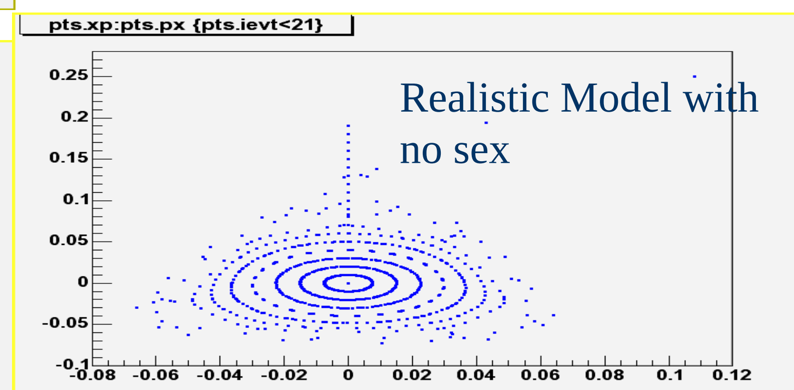

My Hardedge model




Realistic Field Model



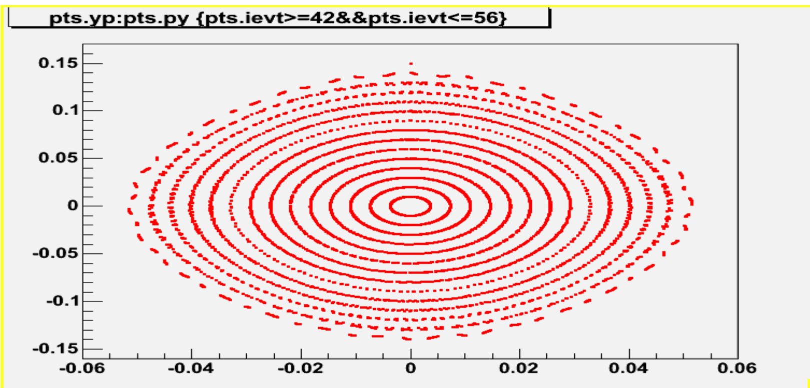

Realistic Model with no sex



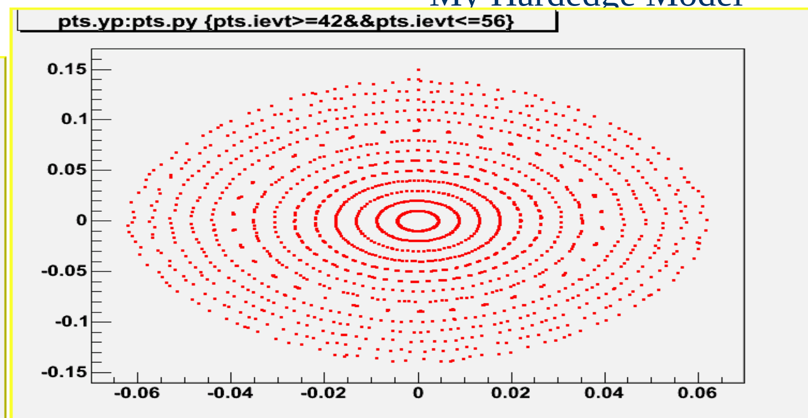
# Vertical Dynamic Aperture (y vs. $p_y$)

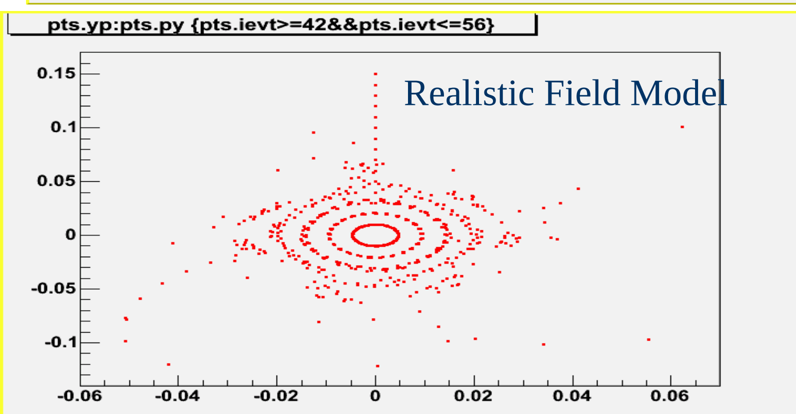
Muon Cooling Ring

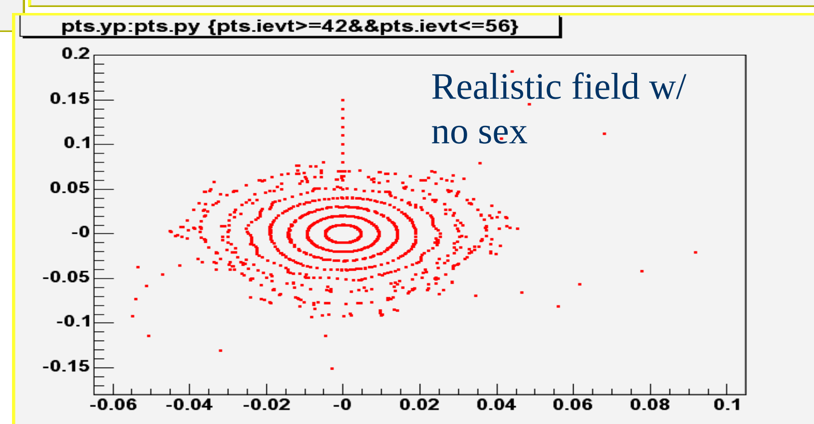
Kirk's Hardedge Model




My Hardedge Model



# Specifications of Triplet Parameters

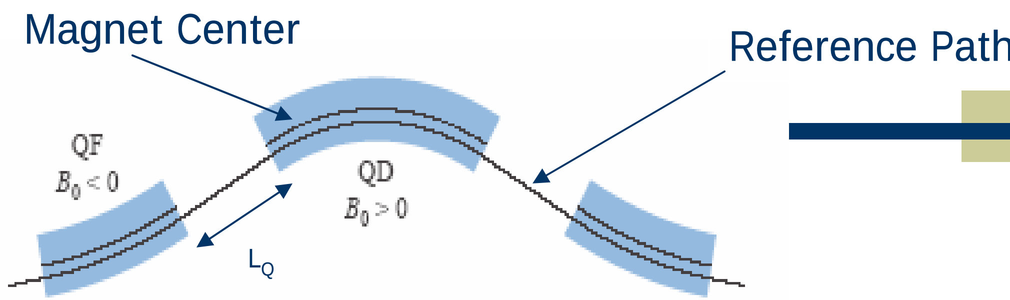
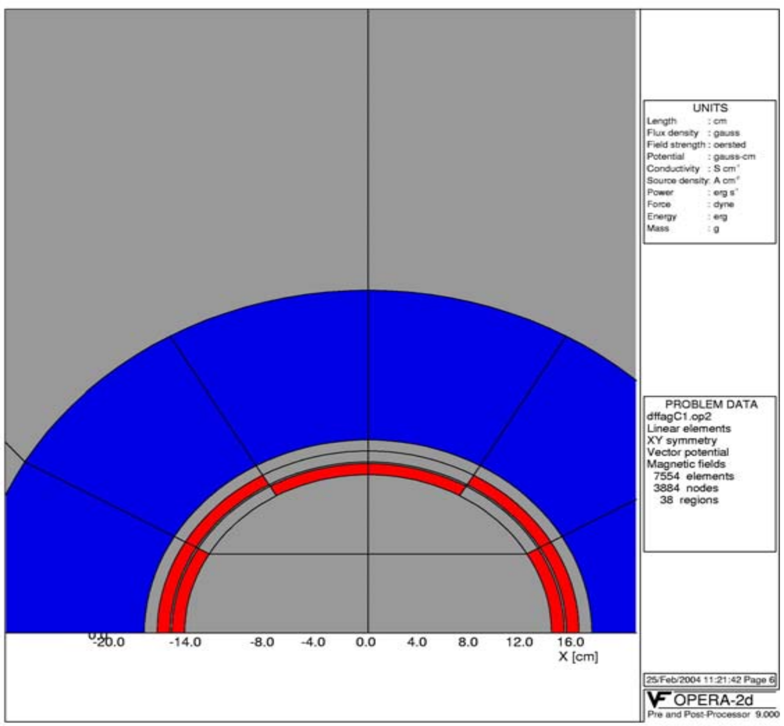



Figure 1: Geometry of the triplet. The displacements of the magnet centers for all magnets are positive. The solid line is the reference orbit, the dot-dashed line goes through the center of the magnet aperture.

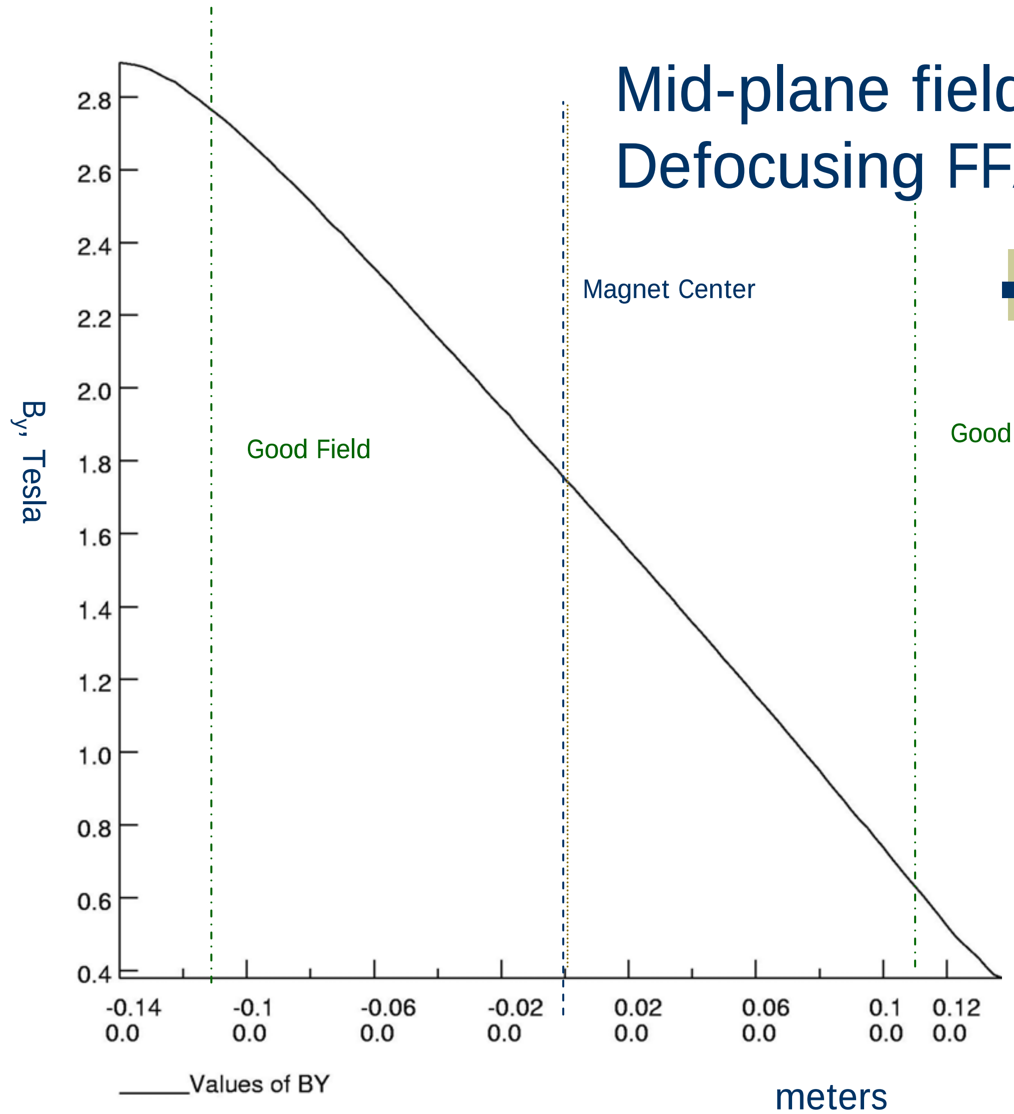
| | | | | |
|---|---|---|---|---|
| $E_{mean}$ (GeV) | 7.5 | | 15 | |
| $E_{min}$ (GeV) | 5 | | 10 | |
| $E_{max}$ (GeV) | 10 | | 20 | |
| $L_0$ (m) | 2 | | | |
| $L_Q$ (m) | 0.5 | | | |
| Numb of Cell | 90 | | 105 | |
| | QD | QF | QD | QF |
| $L_{eff}$ (m) | 1.612338 | 1.0656 | 1.762347 | 1.275747 |
| ρ (m) | 15.274 | -59.6174 | 18.4002 | -70.9958 |
| $x_0$ (mm) | -1.573 | 7.667 | 1.148 | 8.745 |
| $R_{aper}$ (cm) | 14.0916 | 15.2628 | 10.3756 | 12.6256 |
| $B_0$ (T) | 1.63774 | -0.41959 | 2.71917 | -0.70474 |
| G (T/m) | -9.1883 | 8.1768 | -15.4948 | 12.5874 |

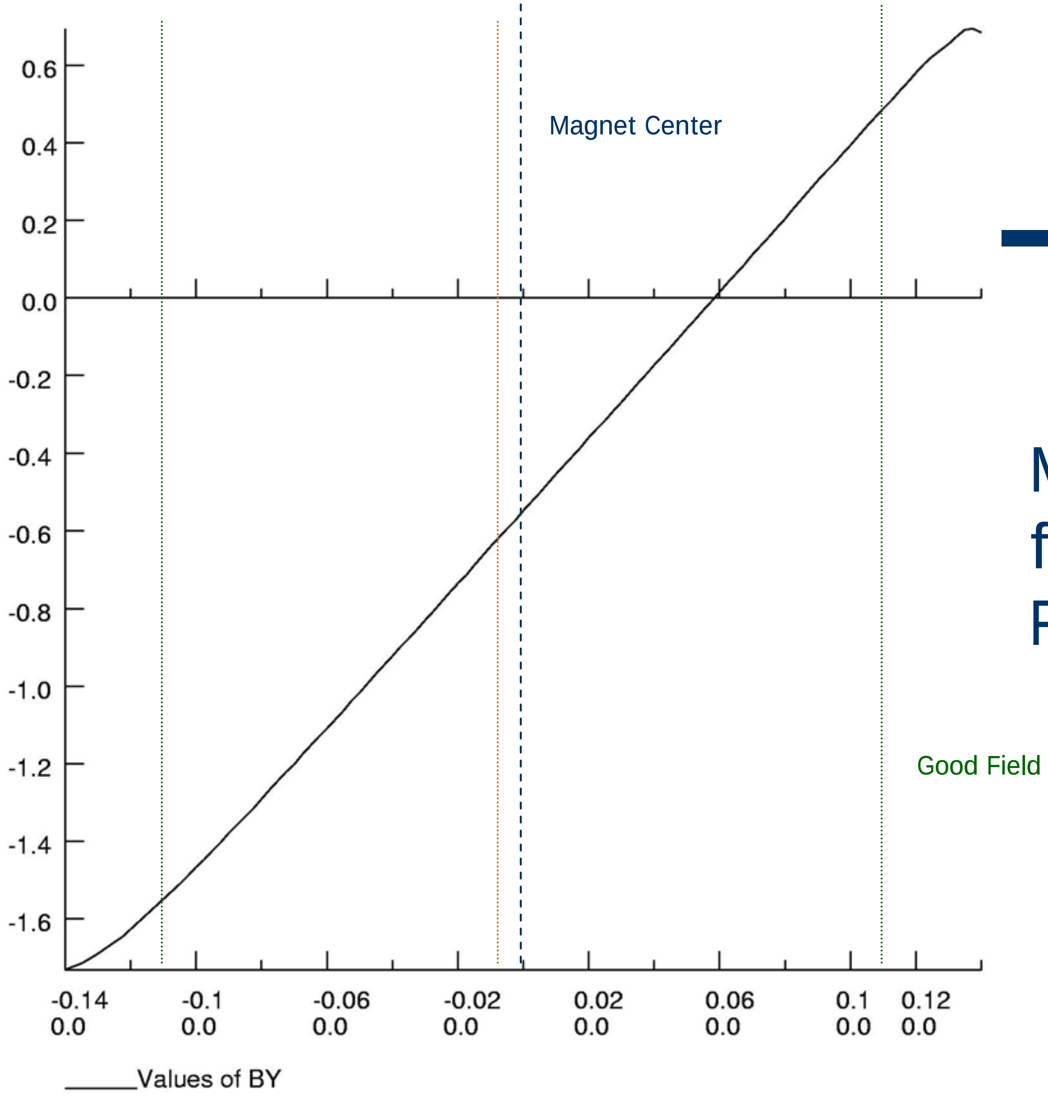
# 2D Cross Section of Magnet Model




- The figure shows the 2D cross section of a magnet that has both dipole and quadrupole coils.
- The yoke is just a simple iron annulus. In construction it would look more like the KEK FFAG cross section

# Mid-plane field for Defocusing FFAG

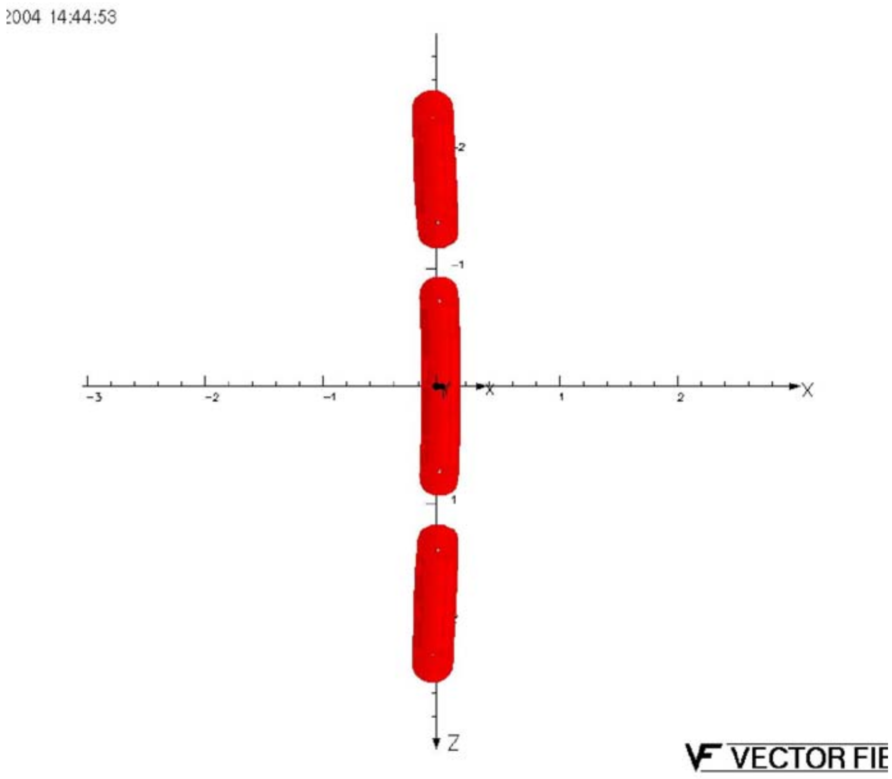




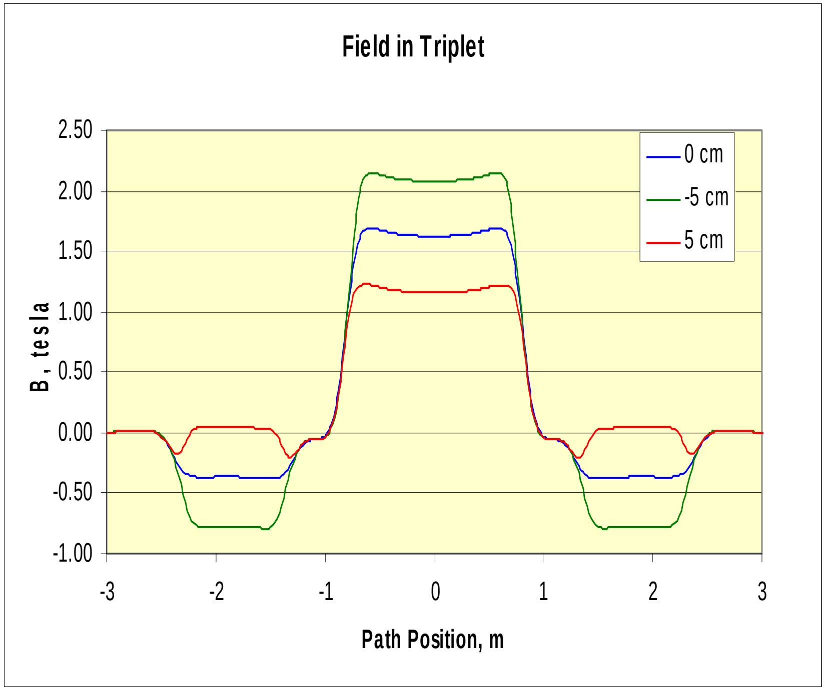

# Mid-Plane Field for the Focusing FFAG

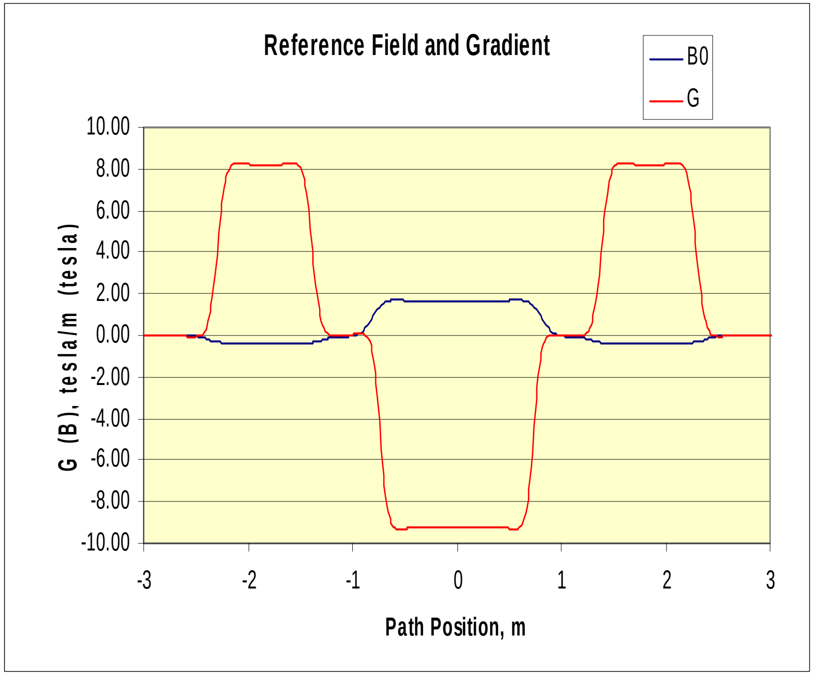
# The Three Dimension Model

- As a first step we shall make a 3D model with TOSCA but without iron.
  - This will be done with iron in the future.
  - We are concentrating on the 5-10 GeV energy range.
  - Current in coils are increased to supply the same $B_0$ and G.
  - The model does take into account the sagitta of the reference path.
    - The QD magnet has a 2 cm sagitta.



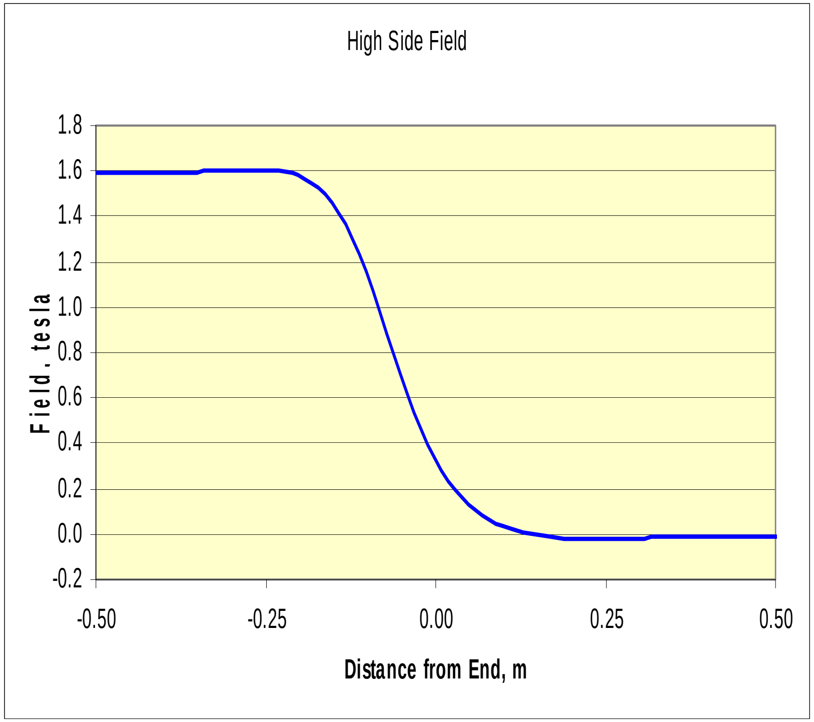
# Field and Gradient along the Reference Path



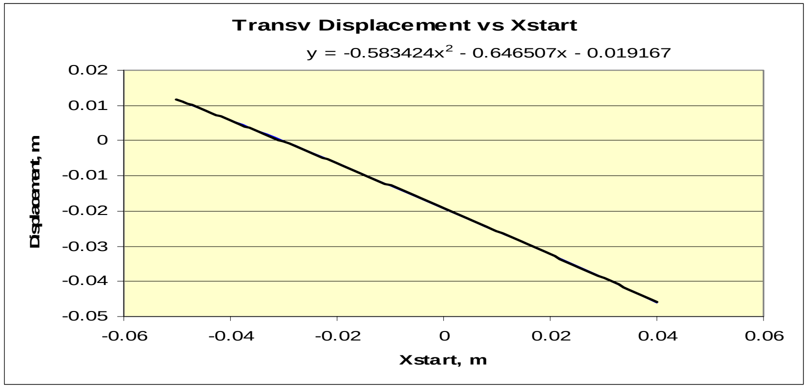
# Triplet End Fields

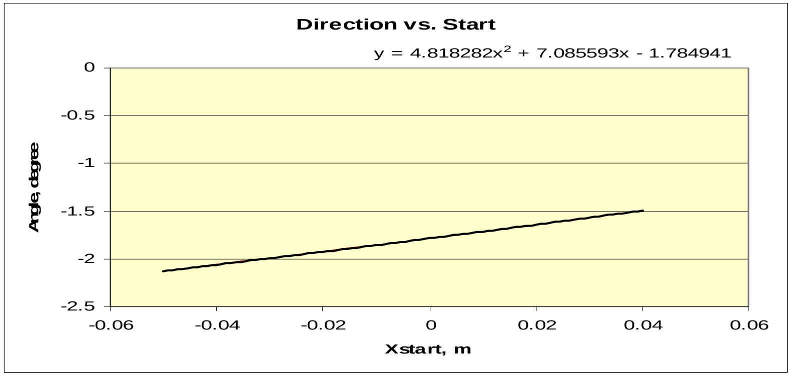
- There will be super-conducting cavities in the space between triplets.
  - The cavities can tolerate a maximum of 1000 kG if the cavities are turned on before the magnet field is.
- The figure shows the end fields along the high field path.
  - The field drops to 1000 kG in ~15 cm from the effective end. Less from the physical end.



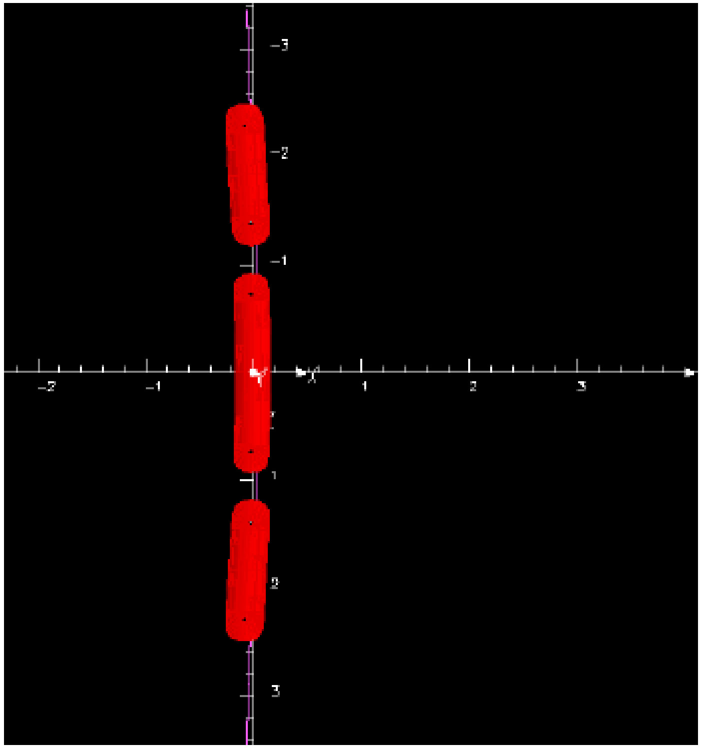
# Finding the Closed Orbit

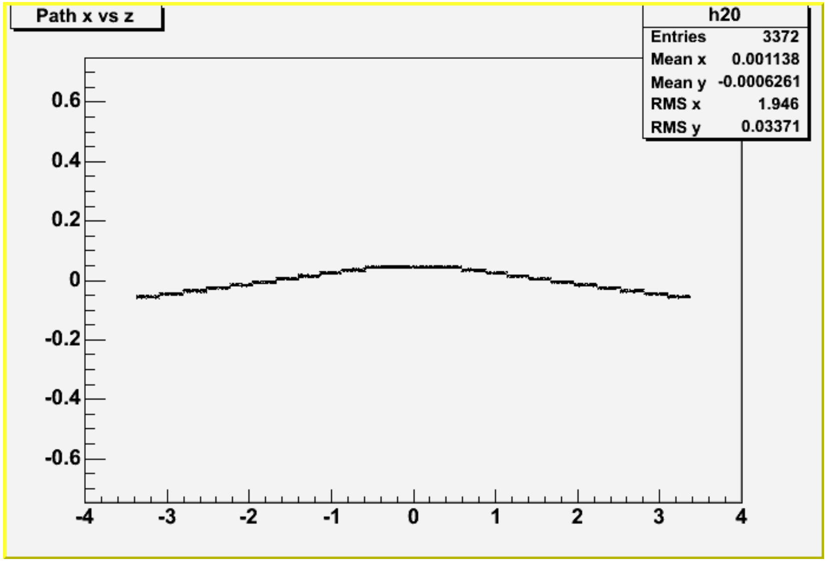
- The closed orbit can be found by tracking particles through one cell from symmetry point to another.
- The top figure shows the *x* deviation after one cell for various start positions of a 7.5 GeV/c muon.
  - This occurs for $X_{start}$=0.02975 m.
- The bottom figure shows the angle after one cell for a 7.5 GeV/c muon. It should be $-2^0$ for a closed orbit.
  - This occurs at $X_{start}$=0.03049 m.
- The difference between these numbers is an indication of the error in this process.



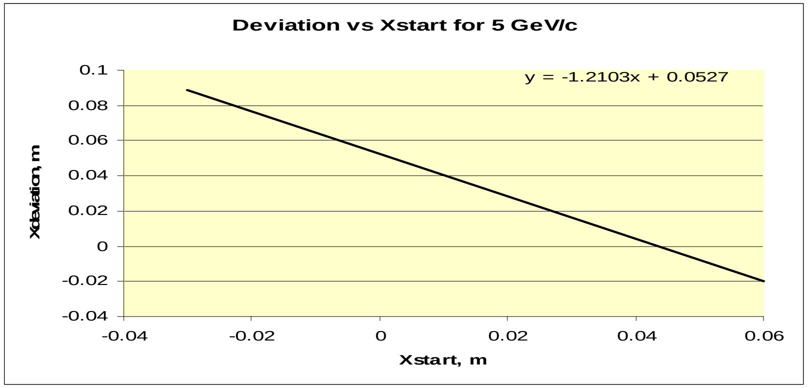
# The 7.5 GeV Path

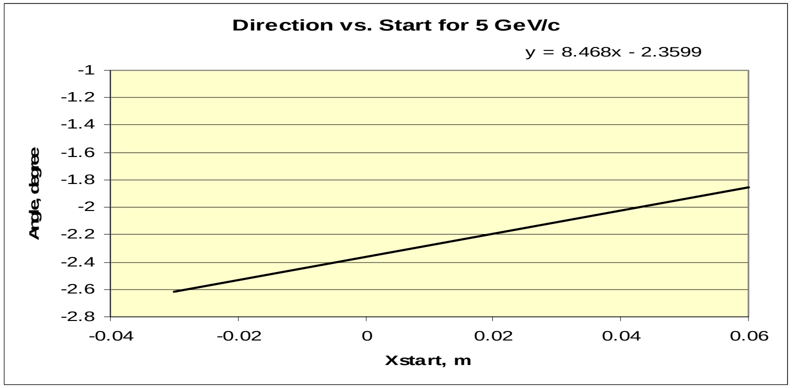




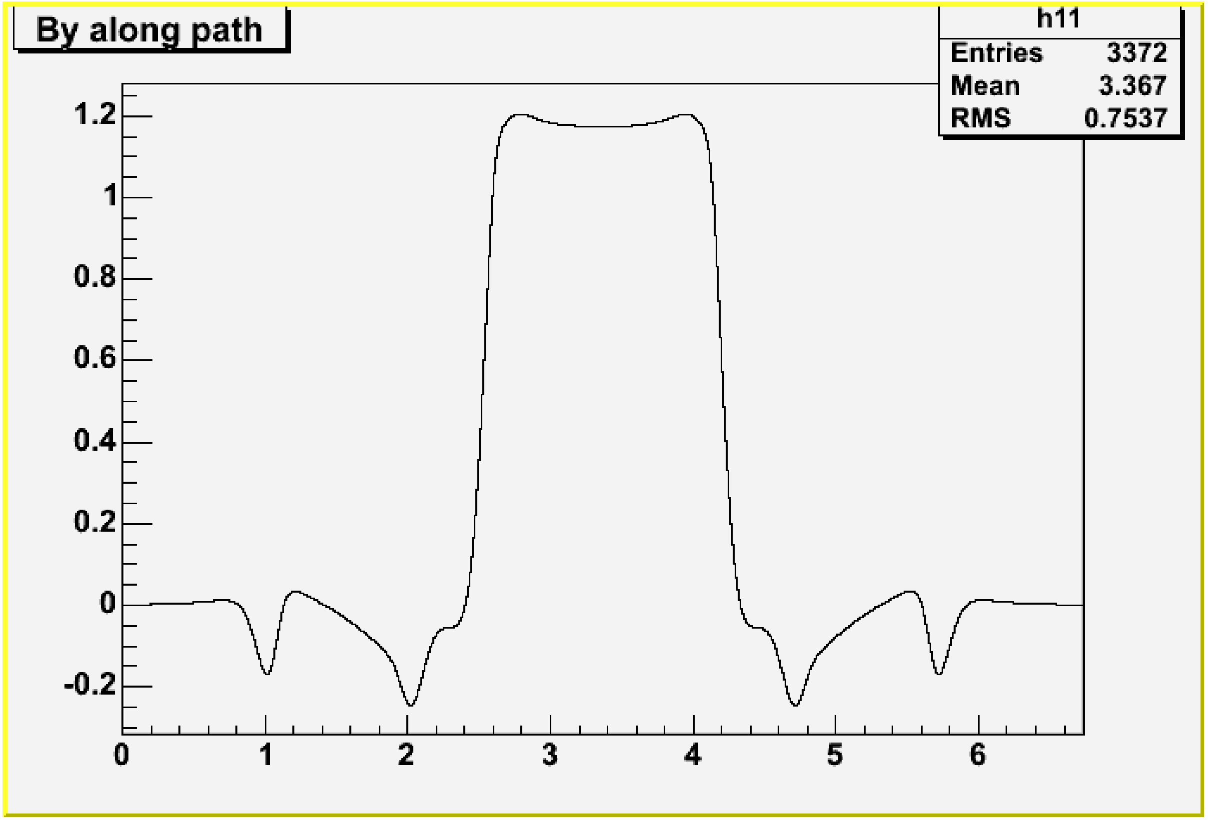
# Closed Orbit for 5 GeV/c

- The closed orbit is a function of the muon momentum. The closed orbit calculated for 5 GeV muons gives:
  - For $X_{deviation}$, $X_{start}$ =0.043543
  - For the direction, $X_{start}$=0.042501 m.

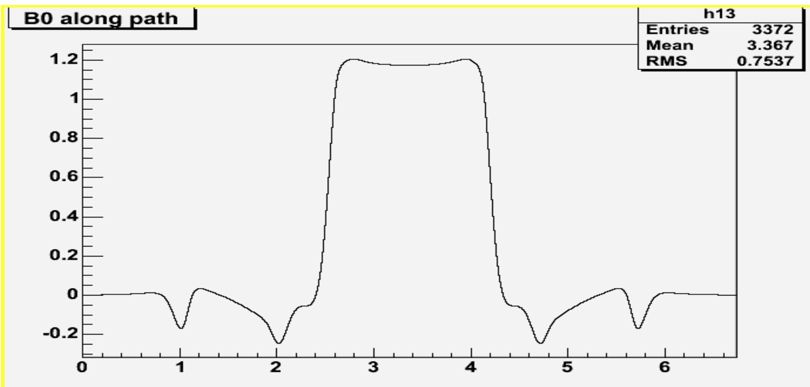


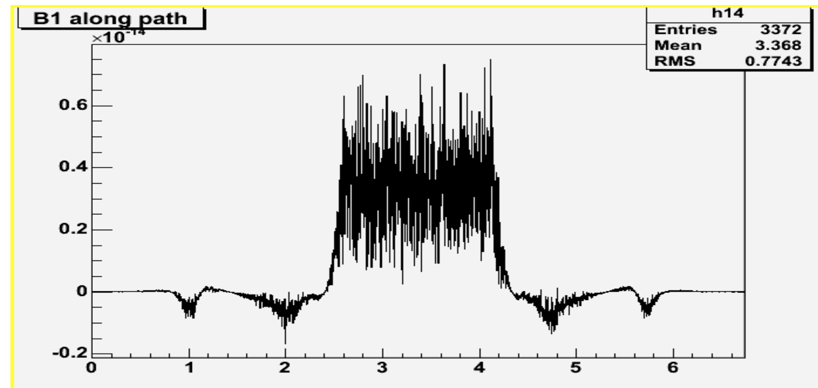



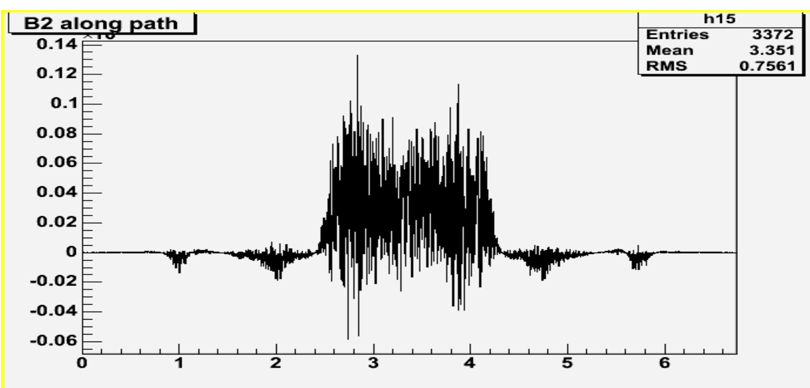
# The Field Along the 7.5 GeV Path

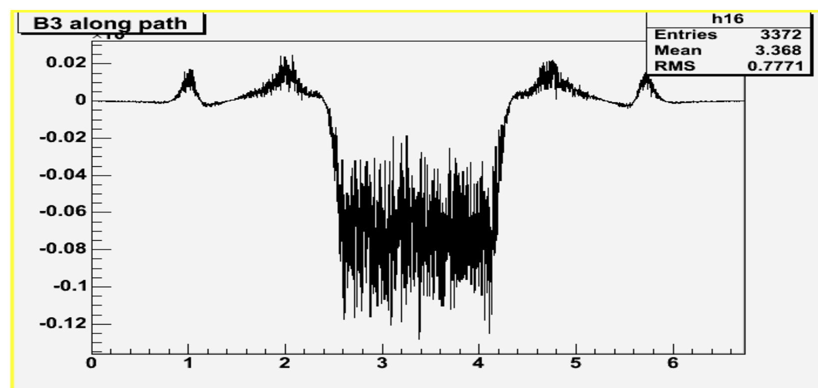


# Mid-plane Harmonics of Field along Path



# Where Do We Go?

- We have a field map
  - We could use this for GEANT.
- We have a closed orbit path.
  - We have the field.
  - We can make a Fourier decomposition of the field along the path.
    - This would give us all the derivitives for ICOOL.
- This should provide us with the tools to evaluate the dynamic aperture of this FFAG accelerator ring.