Electron Models for Muon Acceleration in FFAG Rings

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My WWW home directory:

`http://keil.home.cern.ch/keil/MuMu/Doc/FFAG_Apr04/talk2.pdf`

Motivation of Electron Model

- Demonstrate novel features of acceleration in non-scaling FFAG rings at fraction of cost of FFAG rings in neutrino factory
  - Acceleration outside buckets
  - Crossing of many integral and half-integral resonances
- Electron models
  - accelerate from about 10 to about 20 MeV
  - have focusing by doublets or triplets
  - fit into a small hall
  - are constructed next to a suitable electron linac

Longitudinal Dynamics

- Longitudinal Hamiltonian for stationary buckets

$$H_1(p_t, \varphi) \quad = \quad \frac{\pi h \beta_r^2 E_r}{eVN_c} \left(\frac{\eta_0 p_t^2}{2} + \frac{\eta_1 p_t^3}{3} + \ldots \right) + \sin^2 \pi\varphi$$

  - p_t momentum error relative to reference particle with total energy E_r and speed $\beta_r c$
  - φ phase measured in cycles with origin at stable fixed point and $-1/2 \leq \varphi \leq +1/2$
  - h harmonic number, V peak accelerating voltage, N_c number of RF cavities
- Consider motion near transition with $\eta_0 = 0$, and $\eta_1 \neq 0$

Longitudinal Motion Near Transition

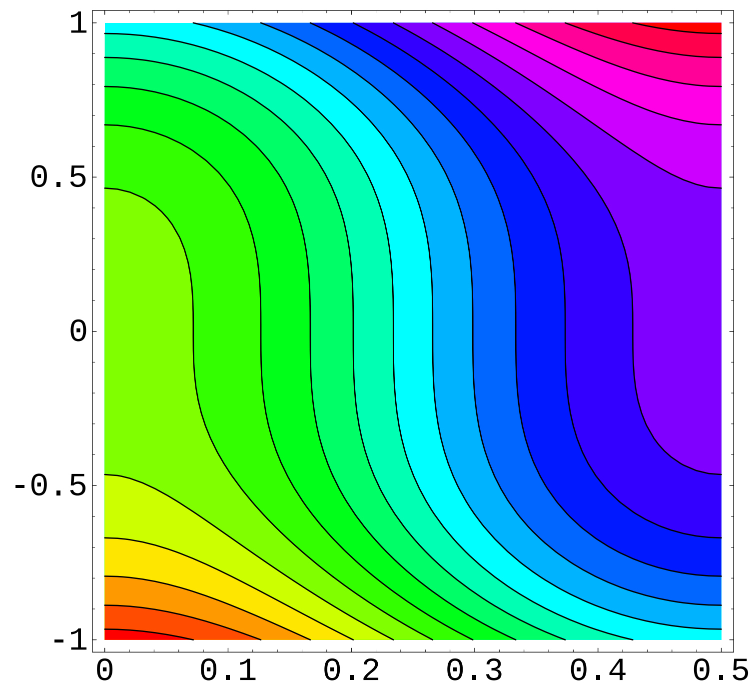
- Introduce scaled momentum variable y

$$y = p_t \left(\frac{2\pi\beta_r^2 E_r h \eta_1}{3eV N_c} \right)^{1/3}$$

- Scaled Hamiltonian $H_5(y, \varphi)$

$$H_5(y, \varphi) = y^3 + \sin^2 \pi\varphi$$

- Acceleration in FFAG rings happens along light blue S-shaped trajectory, which starts at $\varphi = 1/2$ and $y = -1$, and reaches maximum $y = 1$ at $\varphi = 0$
- Equation relates range $\pm p_t$ and ring parameters at $y = \pm 1$, cf. next page
- Apply later to electron model



Contour plot of $H_5(y, \varphi)$

Parameters and Scaling Laws

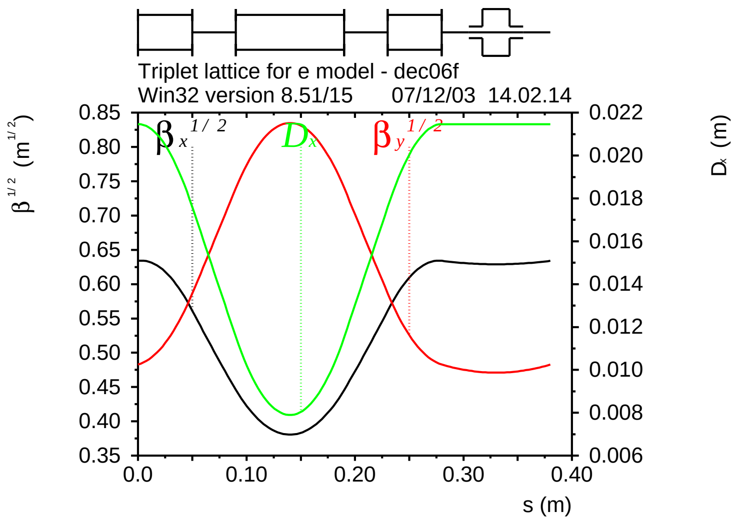
- Calculate RF cavity voltage V from accelerating range p_t and ring parameters:

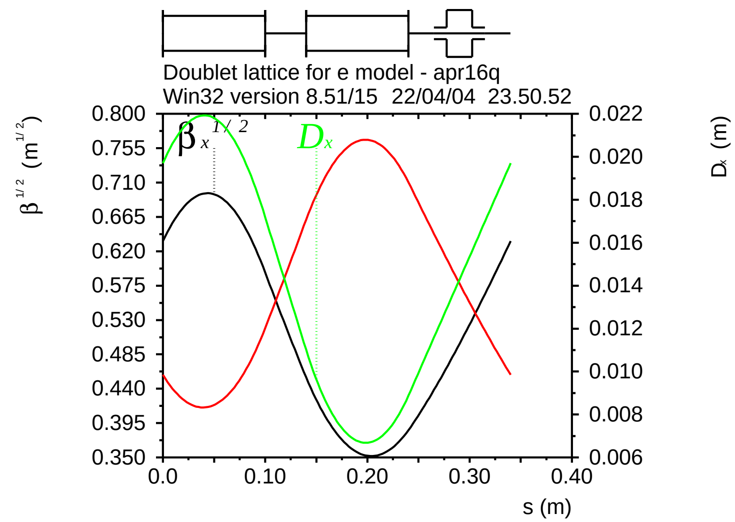
$$V = \frac{2\pi\beta_r^2 E_r h \eta_1 p_t^3}{3eN_c}$$

- Obtain scaling laws for V in ring with N lattice periods of length L:
  - Circumference $C \propto LN$
  - $h \propto LN$ at given RF frequency
  - $N_c \propto N$
  - $\eta_1 \propto 1/N^2$ was derived analytically by K.Y. Ng for FODO lattice with $N \gg 1$; from numerical studies I believe that it holds for any lattice style
  - $V \propto L/N^2$ and $N_c V \propto L/N$
- Assuming that cost of magnets, vacuum, tunnel is $\propto LN$, and that cost of RF cavities and RF power installation is $\propto NV \propto L/N$ yield cost optimum when the two cost components are equal

Electron Model Lattices

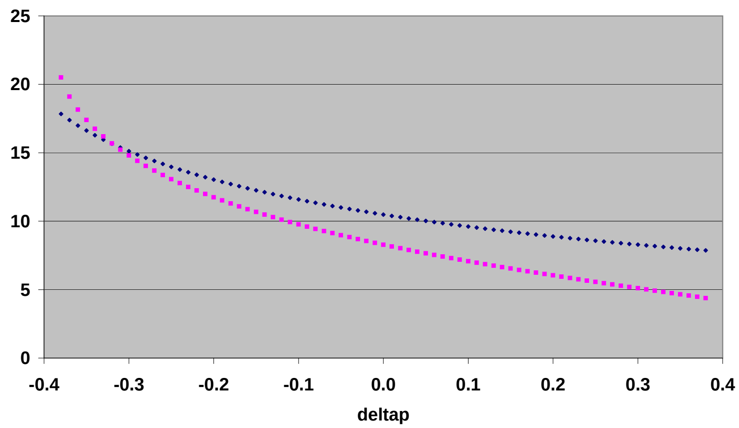
- Triplet lattice above
- Doublet lattice below
- Displaced F and D quadrupoles
- F magnets bend away from ring centre
- Space for coils between magnets
- Comparable aperture and effective length
- Magnetic field within reach of permanent magnets
- Space for room-temperature single-cell RF cavity at 3 GHz, similar to buncher cavity in S-band linac



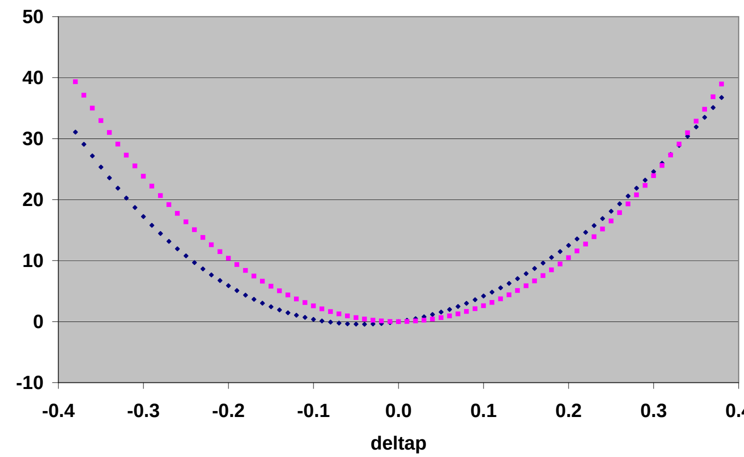


Triplet Figures

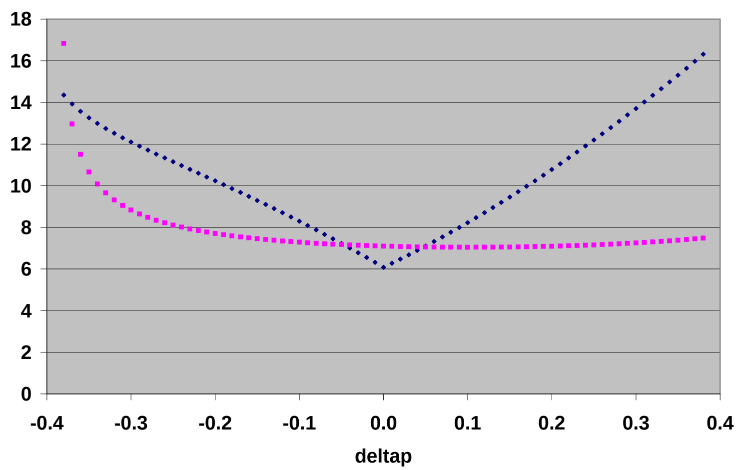
Tunes Q_x and Q_y vs. $\delta p/p$



Path length $\delta(s)$ and travel time ct in mm



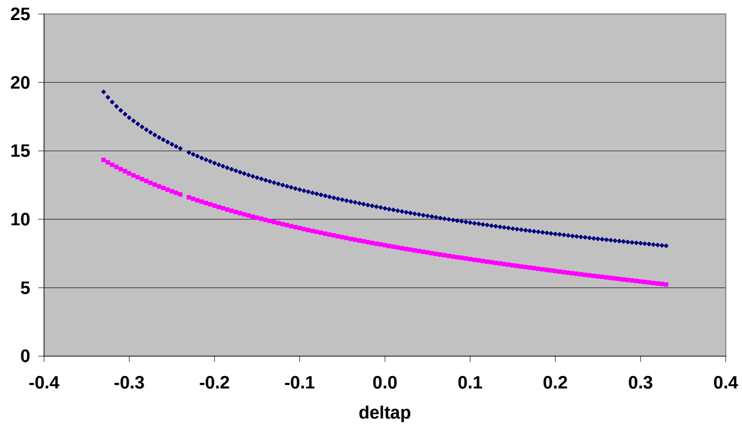
Half apertures A_x and A_y in mm vs. $\delta p/p$



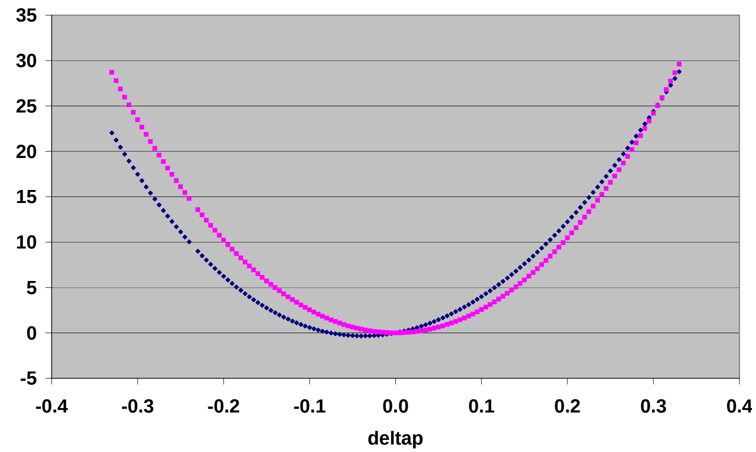
- Stable tunes q_x and q_y in range $-1/3 \leq \delta p/p \leq 1/3$
- $A_x < 20$ mm and $A_y < 10$ mm for $\varepsilon_n = 0.3$ mm, geometrical mean between ATF and CLIC drive beam linacs
- Fit to ct yields $\eta_1 = 0.0149$

Doublet Figures

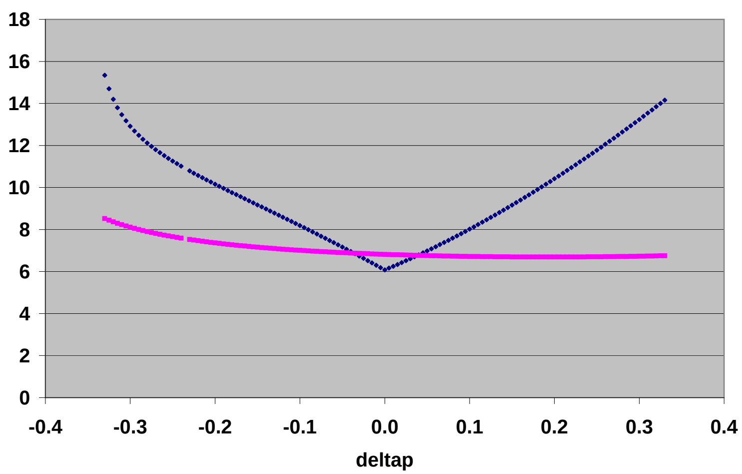
Tunes Q_x and Q_y vs. $\delta p/p$



Path length $\delta(s)$ and travel time ct in mm



Half apertures A_x and A_y in mm vs. $\delta p/p$



- Stable tunes q_x and q_y in range $-1/3 \leq \delta p/p \leq 1/3$
- $A_x < 20$ mm and $A_y < 10$ mm for $\varepsilon_n = 0.3$ mm, geometrical mean between ε_n in ATF and CLIC drive beam linacs
- Fit to ct yields $\eta_1 = 0.0167$

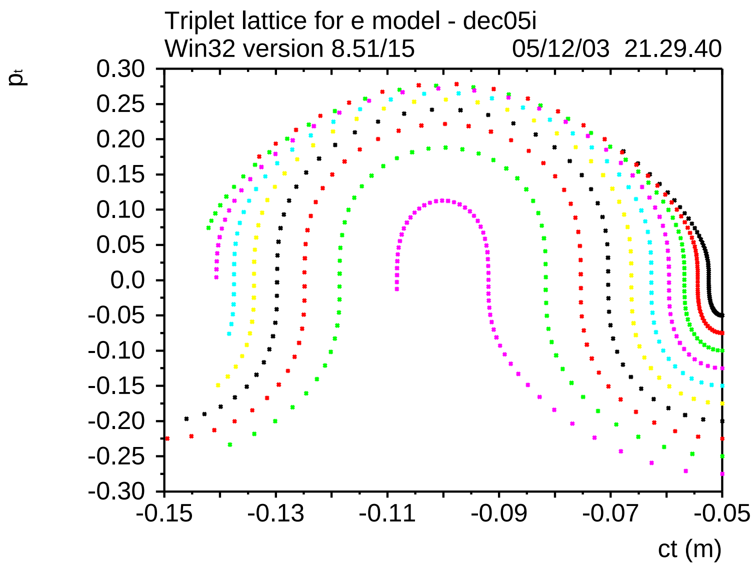
Lattice Parameters at 15 MeV

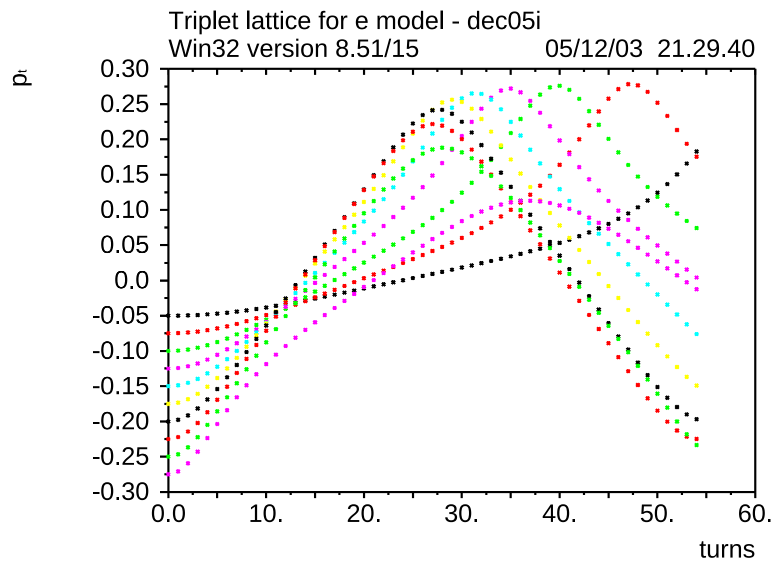
	Triplet	Doublet	
Cell length	0.38	0.34	m
F/D magnet length	50/100	50/100	mm
F/D magnet bore radius	25/32	23/29	mm
F magnet angle	-37.459	-54.333	mrad
F magnet gradient	5.638	5.030	T/m
F magnet central field	-37.464	-27.170	mT
F magnet pole tip field	0.14	0.11	T
F magnet char. length	-6.64	5.40	mm
D magnet angle	214.545	193.959	mrad
D magnet gradient	-4.746	-4.313	T/m
D magnet central field	107.285	96.699	mT
D magnet pole tip field	0.15	0.13	T
D magnet char. length	-22.6	22.4	mm

Comments on Lattice Parameters

- Both lattices have 45 cells
- Char. length $X = B/G =$ quadrupole displacement
- $X_F < A_x \rightarrow$ displaced F quadrupoles
- $X_D > A_x \rightarrow$ displaced half D quadrupoles
- Obtain pole tip field assuming that hyperbolic pole tip passes through corner of rectangular aperture
- Good field region needed extends further between poles than bore radius $\rightarrow$ increase bore radius and pole tip field?
- Product $\eta_1 Q_x^2 = 1.64$ for triplet lattice, 1.95 for doublet lattice; Q_x taken at $\delta p/p = 0$, i.e. at 15 MeV

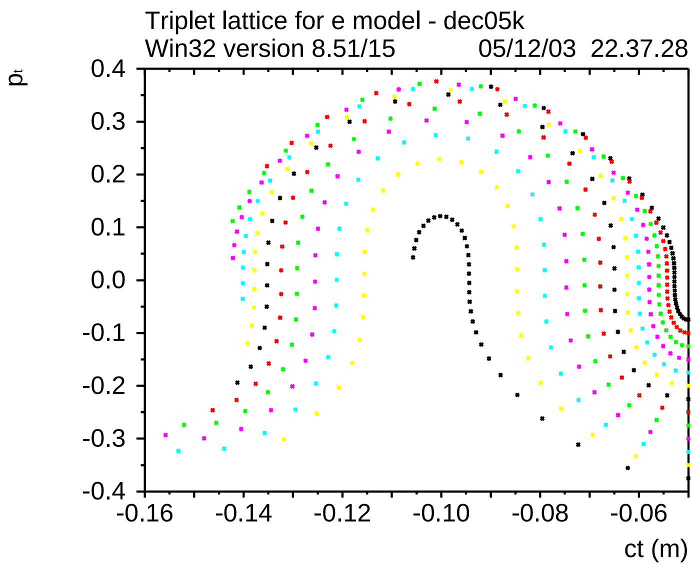
Acceleration in Perfect Triplet Lattice at $V = 20$ kV

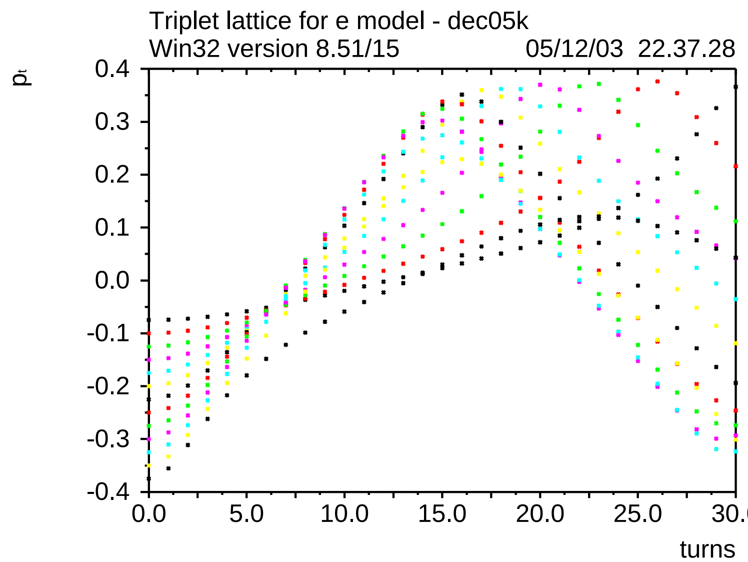




- (ct, p_t) phase space on the left, p_t vs. turn on the right
- Coordinates recorded every 1/5 turn
- Particles launched near $p_t = -0.2$ accelerated to $p_t \approx 0.2$

Acceleration in Perfect Triplet Lattice at $V = 50$ kV





- (ct, p_t) phase space on the left, p_t vs. turn on the right
- Coordinates recorded every 1/5 turn
- Particles launched near $p_t = -0.3$ accelerated to $p_t \approx 0.3$

Beam Loading

- Compare energy extracted by beam W_e to stored energy W_s in cavity:

$$W_s = \frac{U^2}{4\pi f_{\mathrm{RF}}(R/Q)}$$

- Peak cavity voltage $U = V\pi/2$, frequency of RF system $f_{\mathrm{RF}} \approx 3$ GHz, intrinsic impedance $R/Q = 121\Omega$ in pillbox cavity
- With beam current I, acceleration in n turns, W_e becomes with circumference C:

$$W_e = \frac{ICVn}{c\beta_r}$$

- Taking $W_e/W_s \ll 1$ yields upper limit for I with harmonic number h of RF system:

$$I \ll \frac{\pi V}{16nh(R/Q)}$$

- Accurate calculation of transient beam loading should take into account variation of phase and acceleration.
- Beam observation system must work to expected accuracy at beam current I

RF System Parameters

- RF power

$$P = \frac{U^2}{2Q(R/Q)}$$

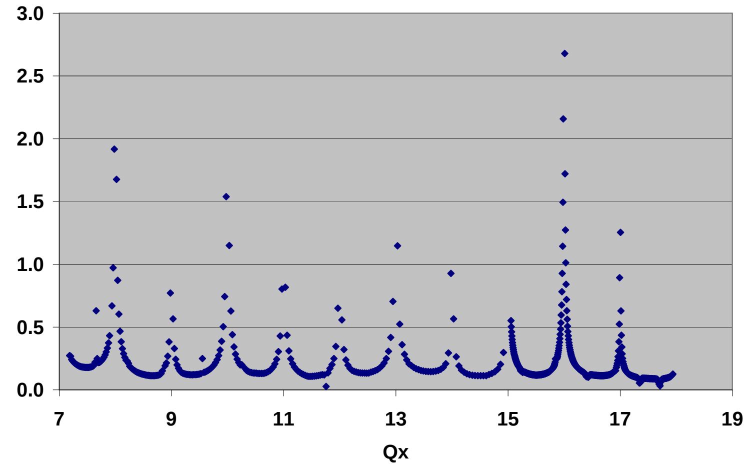
- Quality factor $Q = 17700$ for Cu cavities with $\sigma = 5.7 \times 10^{-7}\Omega^{-1}\mathrm{m}^{-1}$
- Feed with one TWT and one waveguide, tapping off power

	Triplet	Doublet	
Slip factor η_1	0.0149	0.0167	
Number of cavities N_c	45	45	
Harmonic number h	171	153	
Cavity accel. voltage V	50	66	kV
Peak cavity voltage U	78.5	103.7	kV
RF cavity power P	1440	2509	W
Stored energy W_s	1.35	2.35	mJ
Range p_t in formula	0.304	0.335	
Initial p_t	-0.3011	-0.3300	
Final p_t	0.3012	0.3300	
Number of turns n	5	4	
Beam current I	$\ll 94.9$	$\ll 175$	mA

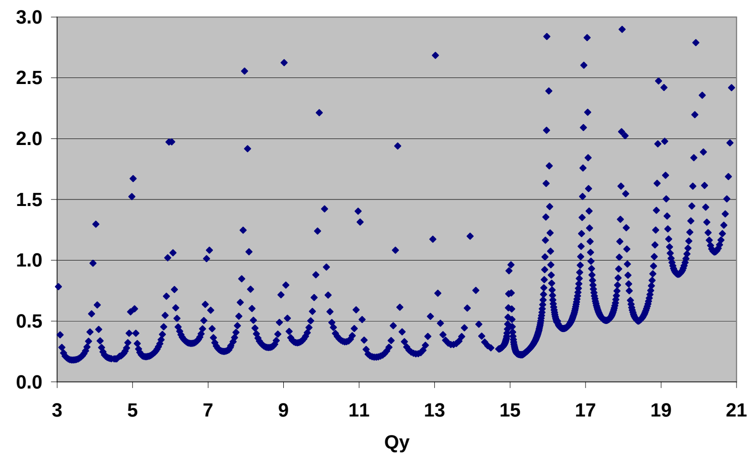
Misalignments in Triplet Lattice

- Triplets perfectly aligned on girders
- Small RMS displacement of girders 0.03 mm, achieved by survey with central monument
- Misalignments drive orbit distortions
- Offsets > 3 mm not shown
- Misalignments also strongly drive D_x and D_y at natural chromaticity, find $D_x = 23.6 \pm 0.7$ mm and $D_y = 8.9 \pm 4.2$ mm
- With gradient errors would get half-integral resonances
- Beam does not circulate at constant $\delta p/p$ close to integral resonances
- Will beam be accelerated across resonances?

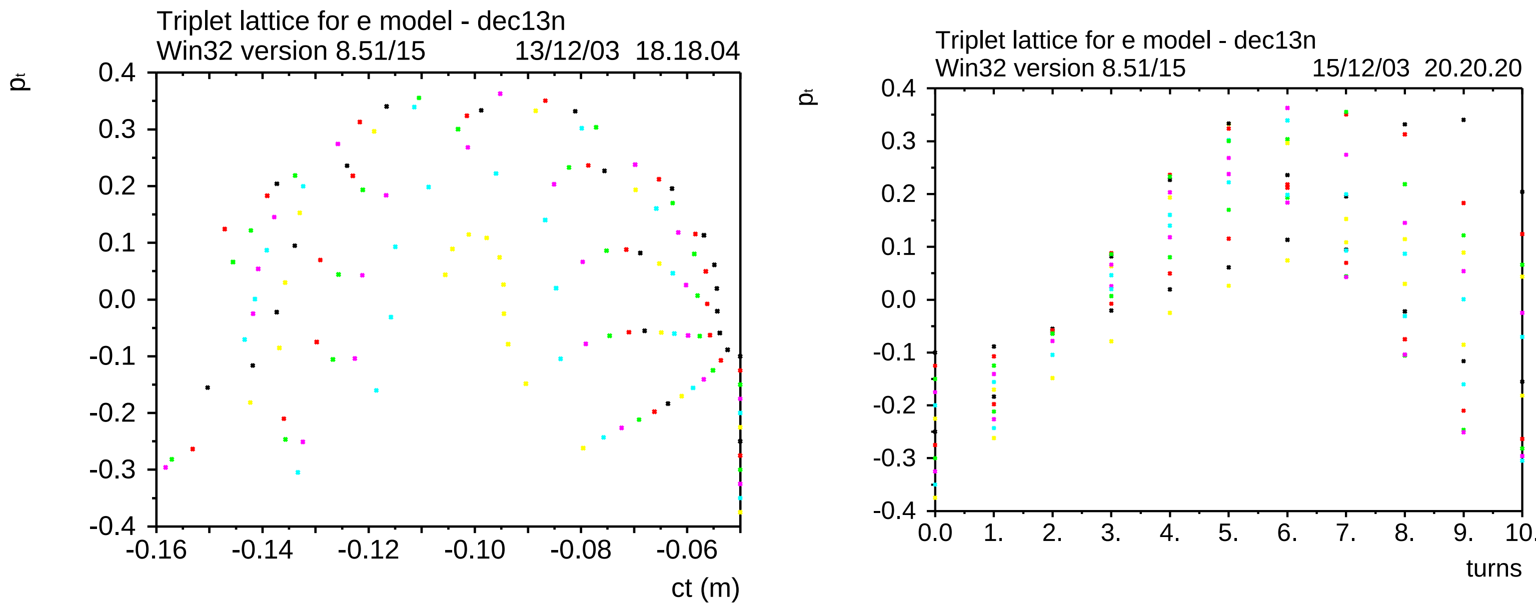
Maximum horizontal orbit offset in mm



Maximum vertical orbit offset in mm

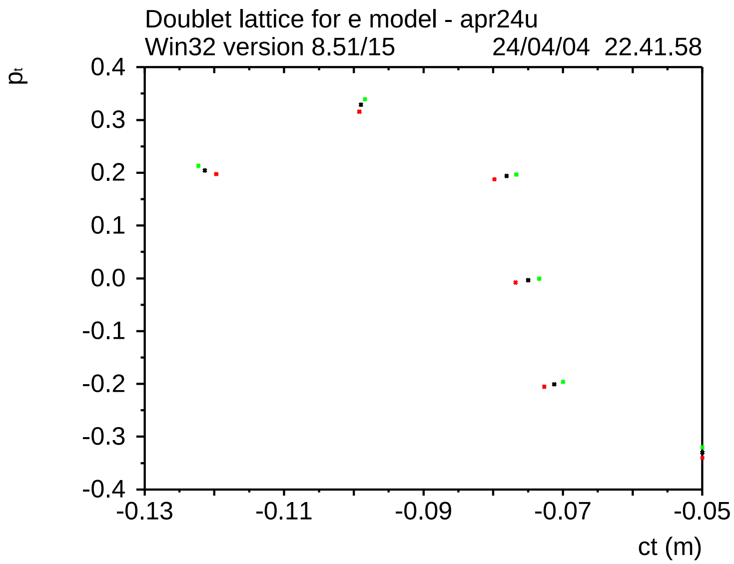


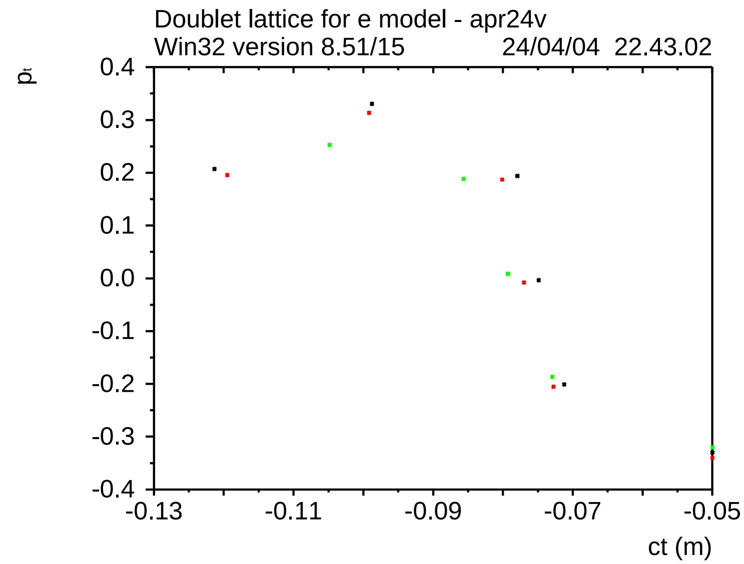
Acceleration in Triplet Lattice with Misalignments at $V = 50$ kV



- (ct, p_t) phase space on the left, p_t vs. turn on the right
- Particles launched near $p_t = -0.15$ and $p_t = -0.3$ are lost

Acceleration in Doublet Lattice at $V = 66$ kV





- (ct, p_t) phase space plots
- Perfect machine on the left, machine with 0.03 mm RMS displacement on the right
- Particles launched at $p_t = -0.34, -0.33, -0.32$
- Green particle lost during turn no. 5